Discrete element modeling of strata and surface movement induced by mining under open-pit final slope

Nengxiong Xu\textsuperscript{a,}\textsuperscript{*}, Jiayin Zhang\textsuperscript{a}, Hong Tian\textsuperscript{b}, Gang Mei\textsuperscript{a}, Quan Ge\textsuperscript{a}

\textsuperscript{a} School of Engineering and Technology, China University of Geosciences, Beijing 100083, China

\textsuperscript{b} School of Engineering, China University of Geosciences, Wuhan 430074, China

\begin{abstract}
Ore mining under the open-pit final slope will result in caving and large displacement movement of rock-masses surrounding the goaf. It is difficult for numerical methods that equate jointed rock-masses to a continuum to simulate the aforementioned mechanical behavior of rock-masses. However, the placing of joints in a calculation model according to their actual distribution characteristics will result in an excess of model blocks, which renders the calculation impossible. To solve this problem, a numerical modeling method based on an Equivalent Jointed Rock-mass Model (EJRM) is proposed in this paper. First, the rock-masses in the research area are simplified as an EJRM; then, a back analysis method, combining orthogonal design and numerical simulation with Discrete Element Method (DEM), is used to invert the mechanical parameters of the model; lastly, the DEM is utilized to analyze the movement and failure of the strata and surface induced by mining under open-pit slope. The Yanqianshan iron ore mine was treated as an example in the present study. Mining under the eastern slope of the open pit of the mine was simulated by means of the aforementioned method. The patterns of the failure and movement of the strata and slope at different mining stages were obtained. This numerical modeling method proposed can be used to simulate complex mechanical behaviors of rock-masses induced by underground mining.
\end{abstract}

\section{Introduction}

The ore bodies under the open-pit final slope are those which lie above the bottom of the mine pit and extend a certain distance from the pit slope to the mine border. To maximize the resource exploitation and increase economic efficiency, the ore bodies under final slope should be first exploited for the subsequent exploitation in the stage of transitioning from open-pit mining to underground mining.

Mining under an open-pit final slope will induce movement and failure of the relevant strata and ground surface,\textsuperscript{1} which may further result in detrimental impacts on adjacent ground infrastructure and thus threaten the safety of production. Especially, when the ore seam thickness is relatively large, mining-induced disturbances will be extremely significant and result in more serious damage to the ground surface and the open-pit slopes. And the ground damage will bring detrimental effects on the ground buildings and infrastructures. To avoid the detrimental effects, it is critical to analyze and predict the movement patterns of the strata and ground surface induced by mining under open-pit slope before and during the mining process.

Limited published literature has, to date, paid attention to strata and surface movement issues related to mining under open-pit slope. Fortunately, we can use the literature about underground mining induced strata movement for reference, because mining under open-pit slope is actually one type of underground mining.

Research on coal mining subsidence prediction has been performed for nearly two centuries. Empirical formulas, curves, graphs and influence functions have been developed based on numerous measurements of mining subsidence.\textsuperscript{2–4} Numerical modeling methods can well analyze the distribution patterns of the stress, strain and displacement of rock-masses during the underground mining process.\textsuperscript{5} Therefore, some researchers have used continuum mechanics-based numerical simulation methods, such as the finite element method (FEM) and finite difference method (FDM) to study the patterns of the movement and failure of the strata and surface induced by underground mining.\textsuperscript{6–8}

However, unlike normal underground excavation,\textsuperscript{9} the mechanical behavior of the strata in response to underground mining is very complex,\textsuperscript{10–12} which significantly hinders the application of numerical simulation methods. First, underground mining causes strong disturbance to the strata, resulting in significant changes in the mechanical characteristics of the strata.\textsuperscript{12} For example,
underground mining directly results in roof caving, and the caved rock-masses becomes loose blocks, which fill the goaf area. There is a significant difference in the mechanical properties of rock-masses before and after caving. Therefore, there are certain defects using a method in which a rock-mass is considered as a continuous elastic-plastic material, and the rock-caving is treated as the yield failure of the material using the elasto-plastics theory. For this reason, when using methods such as the FEM and FDM to simulate underground mining, some researchers first use an empirical formula to determine the range of caved rock-masses and treat the rock-masses as two types of materials with different properties before and after caving to simulate roof caving. In addition, underground mining often results in large deformation, large displacement, discontinuous movement and failure of the surrounding rocks. Thus, mesh-dependent numerical simulation methods, such as the FEM and FDM, will encounter problems such as mesh deformity, which renders the calculation impossible.

The DEM is by nature capable of simulating discontinuous, large displacement motions of rock-masses. Some researchers have simulated underground coal mining using the DEM. DEM-based software UDEC has been used to simulate the mining-induced roof failure and collapse. There is a limitation that fractures could only develop along pre-existing discontinuities. To resolve this problem, Gao et. al. simulated the roof as an assembly of triangular blocks bonded via contacts when evaluating the longwall caving characteristics.

Some researchers study the movement of strata and surface induced by the transition from open-pit to underground mining using numerical simulation methods. The analysis of the block caving-induced failure of the large open pit slopes of the Palabora mine using FEM, DEM, and FDM, respectively, has been described. A huge goaf will be formed when the sublevel caving method is used to mine the ore body under the open-pit slope, and discontinuous, large displacement movements and failure of the rock-masses in the goaf area will inevitably occur. It is difficult to set up such discontinuous surfaces as faults and formation interfaces in the model in advance, as their quantity is usually small. However, in general, there are a very large number of joints stochastically distributed in one study area in reality. Setting up the joints in the calculation model according to the geological conditions will result in an excessive number of blocks in the model, which renders the calculation impossible to proceed. Therefore, jointed rock-masses are equated to a continuum in some studies. However, in this case, failure phenomena (e.g., the caving) of surrounding rocks and discontinuous, large displacement movements of rock-masses are difficult to be simulated. To solve this problem, the present study simplifies the rock-masses in the research area to an Equivalent Jointed Rock-mass Model (EJRM); then inverts the mechanical parameters of the model using an orthogonal design and numerical simulation-combined back analysis method; last, the present study analyzes the patterns of the movement and failure of the strata and surface induced by mining under the open-pit slope using the DEM-based software 3DEC.

2. Methodology

The method proposed in this paper involves mainly three contents: (1) the building of an EJRM for numerical modeling analysis; (2) the inverting of the mechanical parameters for the EJRM; and (3) the analyzing of the mining under final slope induced strata and surface movement using DEM.

2.1. Building an EJRM

2.1.1. Features of an EJRM

How to construct a reasonable calculation model is the key to simulating mining under open-pit slope. This kind of model should be suitable to simulate the behavior (e.g., caving and large displacement movements) of the rock-masses in the overburden strata and the surrounding rocks of the goaf as well as to avoid an overly low calculation efficiency caused by setting up an excessive number of joints in the model.

Generally, researchers simplify a jointed rock mass model as a continuum when analyzing issues with small displacement movements and deformation of rock-masses. But, large-scale continuum modeling where joints can only be accounted for implicitly may not be applicable in analyzing issues with large rock displacement and deformation. The present study simplifies the actual jointed rock-mass model to an EJRM, which is a non-continuum including a certain number of hypothetical joints. An EJRM should satisfy the following conditions:

1. The joint sets in the equivalent model should be formed based on the simplification of the dominant joint sets distributed in the study area.
2. Compared to the scale of the goaf, the block size generated due to the intersecting by joints in the model should have such a moderate scale, that the simulation of the caving and movement of the rock-masses can be realized and the negative impact from an excessive number of blocks on calculation can also be avoided as much as possible.
3. The mechanical parameters of the equivalent model should be determined based on the measured data using the back-analysis method.

Due to the large scale of the calculation model and a large number of joints contained in the calculation model, we treat the rock blocks formed by joint intersection as rigid bodies. Although this simplification ignores the deformation of the rock block itself, it gains two benefits: significantly improved calculation efficiency and a decrease in the number of mechanical parameters required, which eases the determination of the calculation parameters. In the calculation example of the present study, the spacing of joints is very small compared with the scale of the model geometry, and multiple joint sets are considered. In addition, the displacement of a rock block is far greater than the deformation of the rock block itself. Therefore, it is reasonable to treat rock blocks as rigid bodies.

2.1.2. Procedure for constructing an EJRM

To make a calculation model consistent with the actual geological conditions it represents as much as possible, we first employ the geological modeling technology to build a 3D geological model of the study object. Subsequently, we add equivalent joints to the geological model and cut it into a block system, and thus form an EJRM. The detailed steps are as follows:

1. Surface elevation data of the study area is derived from the topographic map, and these scattered 3D heights are used as sampled data.
2. The horizontal projection of the study area is subdivided into a
need to identify the boundary of the area where cracks have appeared to determine the distribution range of the cracks instead of the displacement of the slope becomes dangerous and difficult. Under normal circumstances, it is most suitable to use the observed surface displacement value as the test indicator. However, as mining under open-pit slope results in rock slides, monitoring the displacement of the slope becomes dangerous and difficult. During the underground mining process, deformation and movement of the strata and surface induce cracks on the surface of the slope. Therefore, we use the distribution range of the slope cracks as the test indicator. When conducting the field survey, we only need to identify the boundary of the area where cracks have appeared to determine the distribution range of the cracks instead of measuring the surface displacement in the severely damaged area.

**Step 2.** The joint mechanical parameters, \( kn \), \( ks \), \( C \), and \( \phi \), are determined by combining the back analysis with an orthogonal experimental design technique and numerical simulation. The key idea behind the back analysis is that: first, a multi-factor and multi-level testing program is formed using an orthogonal experimental design technique, where the joint mechanical parameters are selected as the experimental design factors and all the testing schemes are listed in an orthogonal table; then, a series of numerical simulations are conducted using 3DEC with the parameter schemes listed in the orthogonal table; and finally, the discontinuity mechanical parameters are determined according to a range analysis performed on the results and a relation between the test indicator and each of the experimental factors. The detailed procedure is as follows.

**Step 1.** The width of the distribution range of surface cracks (represented with \( W \)) caused by mining is selected as the benchmark value of the test indicator of orthogonal experimental design. Under normal circumstances, it is most suitable to use the observed surface displacement value as the test indicator. However, as mining under open-pit slope results in rock slides, monitoring the displacement of the slope becomes dangerous and difficult. During the underground mining process, deformation and movement of the strata and surface induce cracks on the surface of the slope. Therefore, we use the distribution range of the slope cracks as the test indicator. When conducting the field survey, we only need to identify the boundary of the area where cracks have appeared to determine the distribution range of the cracks instead of measuring the surface displacement in the severely damaged area.
selected as the experimental design factors. In the present study, the rock blocks can just move and rotate, but not deform and fail, because the rock blocks of an EJRM are treated as rigid bodies. The mechanical parameters, such as Yang's modulus, Poisson's ratio, and strength, of rock blocks, will not affect the calculation results in this study. Therefore, only the joint mechanical parameters are selected as the experimental design factors.

**Step 3.** A multi-factor and multi-level testing program is formed using an orthogonal experimental design technique. Here, multi-factor means there exist several experimental design factors; and multi-level means each factor is set different level of values. All the testing schemes are listed in an orthogonal table; and each row of the table represents a testing scheme. Each scheme is actually a combination of a level of factors.

**Step 4.** For each scheme listed in the orthogonal table, a numerical simulation of mining under the open pit final slope is conducted using 3DEC by adopting the factor values of the scheme; and the value of the test indicator, $W$, is figured out. The only difference between any pair of numerical simulations is the different employment of those factor values; and any of other configurations such as the computational model, boundary conditions, and mining scene are the same.

**Step 5.** A range analysis is performed on the numerical simulation results and a relation is established between the test indicator and each of the experimental factors. Analyzing the results by means of range analysis, the joint mechanical parameters are optimized.

2.3. Analyzing strata and surface movement

2.3.1. Calculation method

Mining under the final slope of an open pit will result in a huge goaf and large deformations, large displacement movement and failure of the surrounding rocks and the overburden strata. Continuous medium FEM and FDM cannot simulate this failure mode of the surrounding rocks. Therefore, we selected the three-dimensional (3D) DEM to simulate the patterns of the movement and failure of the strata and slope induced by mining under the final slope of an open pit. We use the 3DEC calculation software in the present study.

2.3.2. Simulation of the mining process

In the present study, underground mining is realized through gradually deleting the ore bodies to be mined in the model. To truly reflect the mining process, the ore bodies in the calculation model are divided into several parts, each of which represents the ore bodies that are mined during a certain period. After each part of the ore bodies is mined, the iterative calculation will be terminated until the impact of carrying out more steps of iteration on the simulation results becomes little enough, and then the values of the movement of the strata and surface are obtained. In this study, when the difference between the two maximum subsidence values in two adjacent 10,000 iterations is less than 1% of one of the maximum subsidence, the iterating will be terminated. For example, the difference between the maximum subsidence value after iterating 140,000 steps and the maximum value after 150,000 steps is less than 1% of the maximum value after 140,000 steps, then the iterating will be terminated.

3. Case study

3.1. Geological and mining background

3.1.1. Geology

The Yanqianshan iron mine is located in Anshan City, Liaoning Province, China. The outcropped strata in the mining area include Anshan Formation, Liaohe Formation, and Quaternary metamorphic rocks, although the rock strata are mainly composed of Anshan Metamorphic rocks. The basic structural pattern in the mining area is a steep monoclinic structure trending in the direction of 270–300°, with a dip direction in the northeast or southwest and a dip angle of 70–88°; that is, the structure is partially upright. The eastern final slope of the Yanqianshan open pit is located in the area from the VIII to IX + 100 prospecting lines; as seen in Fig. 2. In this area, the iron ore band is located in the middle, strikes almost east-west, dips to the northeast at approximately 70–88°, and has a width from 55 m to 194 m from south to north. The southern part of the final slope is mainly distributed with gneissic mixed and granite-like mixed rocks, while the northern part is distributed with carbonaceous phyllite and chlorite schist (Fig. 1(b)).

3.1.2. Rock types and joints

The study area can be divided into three subareas (North, South and Middle) (Fig. 1(b)), and the rock types and joints in each subarea are described as follows:

The north subarea of the study area is a rock formation with carbonaceous phyllite. The rock-masses in this formation have a fragmentized structure and contain strongly weathered zones. There are three dominant joint sets in this rock formation.

The middle subarea of the study area is a rock formation with banded magnetite quartzite. This rock formation is an Anshan-type iron ore layer (ore body) and is primarily composed of magnetite quartzite and amphibole magnetite quartzite. It generally contains three dominant joint sets. The joints mainly are...
primary and tectonic joints, most of them are closed, and some of them are filled with argillaceous materials. They are relatively poorly interconnected and contain a few weak discontinuities. The interlayer bonding is very good, with a little interlayer dislocation and opening.

The south subarea of the study area has a mixed granite rock formation. The rock-masses inside this rock formation have massive and gneissic structures, and significantly affected by the structures, and thus generally are a weathering and unloading zone. Three joint sets exist here. The joints are mainly composed of tectonic joints and unloading and weathered fractures. They are well inter-connected, most of them are open and intercalated with mud whose thickness is generally larger than the undulated height of the joints. Therefore, the joints have weak bite force, and result in a lot of unstable rock blocks in the formation.

The stereographic plots of joint sets in the rock-masses of the

![Stereographic plots of joint sets in the three parts of the study area](image)

**Fig. 3.** Stereographic plots of joint sets in the three parts of the study area (a) North subarea, (b) Middle subarea, (c) South subarea.

<table>
<thead>
<tr>
<th>Rock types</th>
<th>Subarea</th>
<th>Average occurrence Orientation (° dip)</th>
<th>Average spacing (m)</th>
<th>Conditions of joints</th>
<th>Conditions of ground water</th>
<th>RQD values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbonaceous phyllite</td>
<td>North</td>
<td>Set 1: 32°N 66°E Set 2: 22°N 29°E Set 3: 29°N 74°E</td>
<td>0.3</td>
<td>4 - 8 mm Continuous, Smooth</td>
<td>Dripping</td>
<td>24%</td>
</tr>
<tr>
<td>Granite mixed rocks</td>
<td>South</td>
<td>Set 1: 40°N 75°E Set 2: 22°N 20°E Set 3: 30°N 80°E</td>
<td>0.4</td>
<td>4-6 mm, Continuous, Smooth</td>
<td>Dripping</td>
<td>28%</td>
</tr>
<tr>
<td>Magnetite quartzite</td>
<td>Middle</td>
<td>Set 1: 27°N 80°E Set 2: 20°N 28°E Set 3: 30°N 72°E</td>
<td>1.0</td>
<td>&lt;1 mm, Rough, Moderately weathered</td>
<td>Wet</td>
<td>65%</td>
</tr>
</tbody>
</table>
three areas are shown in Fig. 3. The mechanical parameters of the joint sets in the three areas are listed in Table 1. The physical and mechanical parameters of the intact rock and rock-masses of the three rock types are shown in Table 2.

### 3.1.3. Mining

Yanqianshan iron mine was first constructed in 1960, and the open-pit mining was finished in September 2012. An open pit, 1410 m long in the east-west direction and 570 m to 710 m wide in the south-north direction, is formed after the open-pit mining. The elevation of the closed loop of the pit is about 93 m; and that of the ultimate bottom is 183 m. The slope angles are approximately 35°–38°, 32°–35°, 30°, and 43°, on the southern, northern, western, and eastern final slopes, respectively. After the completion of open-pit mining, ore body under the eastern final slope of the mine pit will be exploited. A sublevel caving method, with horizontal slice mining from top to bottom, has been used. The ore body under the eastern final slope is divided from top to bottom into four horizontal layers: the elevation ranges for the first to the fourth layer are 51 m to 69 m, 69 m to 87 m, 87 m to 105 m, and 105 m to 123 m, respectively (Fig. 4). The horizontal mining range for each layer is about 100 m to 140 m long in the east-west direction and approximately 130 m wide in the south-north direction (Fig. 4).

The ore body is mined from the first to the fourth layer; and for each layer, the mining direction is from the east to the west (Fig. 5). It approximately costs one year to finish the mining of each layer. The mining induces the strata movement and caving, and leads surface subsidence (Fig. 4(c)).

### 3.2. Current scenario of eastern final slope deformation and failure

In December 2013, the mining of the first layer under the eastern slope was completed, and the second layer is currently being mined. A landslide occurred on the eastern slope in February 2014. The areas on the eastern slope after the landslide are presented in Fig. 6(a). There were mining-induced cracks on the slope surface; and the landslide occurred above the mined-out area. We investigated the conditions of the landslide by performing a survey over the eastern final slope. The survey region was an approximate rectangle; and the elevation range was 69 m to 65 m. The boundaries on the south and north sides were 200 m from the center of the observed landslide area. For each survey site, we examined the state of destruction of the rock-masses on the slope and measured the width and trending direction of surface cracks. The state of the slope failure at the eastern slope was used to classify the slope surface into four regions; see Fig. 6(a).

1. **Region I** is located directly above the goaf. The rock-masses of the eastern final slope in this region subside, collapse, and slide. The overlying strata of the goaf are relatively thin, therefore, after the ore bodies are exploited, the roof rock collapses, resulting in further subsidence, collapse, and slide of the slope surface.
2. **Region II** is located at the eastern side of the Region I. When the overlying strata on the goaf slide and subside toward the pit, a rock displacement toward the pit in Region II generates tension cracks those are nearly in the south-north direction. The widths of the cracks range from 5 cm to 20 cm.

---

### Table 2
Mechanical parameters of the intact rock and rock-masses.

<table>
<thead>
<tr>
<th>Rock types</th>
<th>Subarea</th>
<th>Uniaxial compressive strength of intact rock (MPa)</th>
<th>E (GPa) of intact rock</th>
<th>μ of intact rock</th>
<th>E (GPa) of rock masses</th>
<th>G (GPa) of rock masses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbonaceous phyllite</td>
<td>North</td>
<td>43.5</td>
<td>5.34</td>
<td>0.21</td>
<td>2.5</td>
<td>1.01</td>
</tr>
<tr>
<td>Granite mixed rocks</td>
<td>South</td>
<td>48.7</td>
<td>6.7</td>
<td>0.2</td>
<td>3.15</td>
<td>1.28</td>
</tr>
<tr>
<td>Magnetite quartzite</td>
<td>Middle</td>
<td>94.5</td>
<td>27.9</td>
<td>0.14</td>
<td>13.4</td>
<td>5.77</td>
</tr>
</tbody>
</table>

---

**Fig. 4.** (a) The I-I geologic section; (b) The cross Section II-II in Fig. 2; (c) The strata movement and caving induced by underground mining.
(3) **Region III** is located at the north side of the Region I. Multiple tension cracks are distributed in parallel in this region; and trending directions of these cracks are approximately $280^\circ$ to $300^\circ$. The widths of cracks range from 1 cm to 15 cm and gradually decrease from south to north.

(4) **Region IV** is located at the south side of the Region I. A few tension cracks are distributed in parallel in this region; and the crack trending directions are approximately $190^\circ$ to $220^\circ$. The crack widths range from 2 cm to 25 cm and gradually decrease from north to south.

**Fig. 6(b)** is the horizontal projection of the distribution of cracks presented in Fig. 6(a), where P1–P8 is the observation sites of typical cracks in each region. The range of cracks appearing on the ground surface of the slope that was observed for the site survey is illustrated in Fig. 6(b). The maximum width of the area with cracks in the south-north direction is approximately 317 m; and the elevation range is between $-65$ m and 35 m.

### 3.3. The computational model

The computational model is an EJRM which is built using the method introduced in Section 2.1. The modeling domain is the model area shown in Fig. 2. The elevation of the model ranges
from the ground surface to a level of −300 m. The model is
1200 m long in the south-north direction and 700 m wide in the
east-west direction (Fig. 1(a)).

It has been observed from Fig. 3 and Table 1 that: there are
three dominant groups of joints in the north, the south, and the
middle of the study area, respectively. And most importantly, the
distributions of those three dominant sets of joints in the above
mentioned three different areas are quite similar in terms of oc-
currence. Therefore, for simplicity and convenience, only three
dominant sets of joints having same occurrences with those dis-
tributing in the middle area are adopted to build the EJRM. The
average orientations and dip angles of the three main groups of
joints are 40° ± 75°, 220° ± 20°, and 300° ± 80°, respectively. The
spacing of joints are changed according to the distance between
the distributing area and the mining area. The spacing of joints is
4 m in the region in the vicinity of the ore body to be mined, and
gradually increases with distance far away from the mining area.

The three groups of joints divide the computational model into
157,019 blocks (Fig. 1(a)). For this model, the positive X-axis of the
coordinate system in the model points toward the east; the posi-
tive Y-axis toward the north; and the positive Z-axis toward the
sky.

The boundary conditions of the model are as follows: (1) the
fixed displacement constraints in the X-direction are applied on
the eastern and western boundaries of the model; (2) the fixed
displacement constraints in the Y-direction are applied on the
southern and northern boundaries; and (3) the fixed displacement
constraint in the Z-direction is applied at the bottom of the model.
In addition, there are no constraints added to the upper surface of
the model; and the gravity of the rock-masses and ore body is
accounted for in the calculation using a gravitational acceleration
of 9.8 m/s².

3.4. Inverting the mechanical parameters of the joints

Using the method and procedures introduced in Section 2.2,
the mechanical parameters of the joints are inverted.

3.4.1. Initial values of the joint properties

As the blocks in the computational model are treated as rigid
bodies in this study, it is unnecessary to consider the mechanical
properties of the rock blocks. Instead, it is only necessary to focus
on the mechanical properties of the joints. The mechanical para-

meters of a joint include normal stiffness ($k_n$), tangent stiffness
($k_s$), internal friction angle ($\phi$) and cohesive strength ($C$).

The study area contains three subareas (North, South and Middle) (Fig. 1), with different rock types. In each subarea, al-
though the properties of each joint set are different from those of
other sets, we assign the same value for each parameter of all the
joints in order to simplify the back analysis. The initial values of
the mechanical parameters of joints in each part are determined
by using the following procedures:

3.4.1.1. Evaluating the RMR values. The RMR values of the rock-
masses of the three subareas are 26, 30 and 55, respectively. These
values are evaluated based on the estimation method of RMR in-
troduced in the literature and the physical and mechanical
parameters of the rock-masses listed in Tables 1 and 2.

3.4.1.2. Calculating the Shear and Young’s modulus of rock-masses.
The Young’s modulus of rock-masses in each subarea is calculated
using the formula (1) introduced in literature. In addition,
the value of Poisson’s ratio for rock mass is approximately 10% to
20% higher than that of the intact rock. The shear modulus of
intact rock and rock-mass are calculated according to the formula
(2).

$$E_r = 10^{6} \frac{RMR - 10}{40}$$  \hspace{1cm} (1)

In the above formula, $E_r$ is the Young’s modulus of rock-mass.

$$G = \frac{E}{2(1 + \nu)}$$  \hspace{1cm} (2)

Where, $G$ is the Shear modulus of intact rock or rock-mass; $E$
is the Young’s modulus of intact rock or rock-mass, and $\nu$ is Poisson’s
ratio of intact rock or rock-mass.

The calculated Shear and Young’s modulus of rock-masses are
listed in Table 2.

3.4.1.3. Determining the initial values of the joint properties.
The values of normal stiffness ($k_n$), tangent stiffness ($k_s$) of
joints are calculated using formula (3) and (4) and are listed
in Table 3.

$$k_n = \frac{E_r E_i}{s(E_i - E_r)}$$  \hspace{1cm} (3)

$$k_s = \frac{G_r C_i}{s(G_i - G_r)}$$  \hspace{1cm} (4)
In Table 3, the initial values of the mechanical parameters of the joints are determined by using direct shear tests.

3.4.2. Back analysis

The initial values of $k_n$, $k_s$, $C$ and $\phi$ of joints in each subarea, listed in Table 3, are not the ultimate parameters for numerical modeling. They are only treated as references for determining (1) the variation range of each parameter of joints, and (2) the ratio of the corresponding values of the same parameters belonging to different types of rocks, e.g., the ratio of two values of $C$ of two rock types. The ultimate mechanical parameters of joints of the computational model are determined by using back analysis with an orthogonal experimental design technique and numerical simulation.

3.4.2.1. Selection of experimental factors. An experimental factor of orthogonal experimental design technique is a variable which can affect the calculation result. In this study, the mechanical parameters of joints, $k_n$, $k_s$, $C$ and $\phi$, can be selected as the experimental factors. It is obviously that there are three subareas in the research area, and the mechanical parameter values of joints in each subarea are different from each other. If $k_n$, $k_s$, $C$ and $\phi$ of all the subareas are used as the experimental factors of the orthogonal experiments, there are 12 factors in total. The back-analysis calculation for such a system is very tedious and difficult. Therefore, the average values, represented as $\bar{k}_n$, $\bar{k}_s$, $\bar{C}$ and $\bar{\phi}$, of $k_n$, $k_s$, $C$ and $\phi$ of all the subareas, are taken as the experimental factors in the orthogonal experiments.

To determine the mechanical parameters of joints in each subarea when the values of $\bar{k}_n$, $\bar{k}_s$, $\bar{C}$ and $\bar{\phi}$ are given, the ratio between the initial parameter value and the corresponding average value is calculated for each parameter of joints in each subarea. These ratios are listed as $\lambda_{i\alpha}$, $\lambda_{i\beta}$, $\lambda_{i\gamma}$, and $\lambda_{i\delta}$, in Table 3, where $i$ represents the $i$th subarea. For example, in Table 3 the parameter $\lambda_{i\phi}$ represents the ratio between the $\phi$ value of the rock masses located in the $i$th subarea and the average value of all three $\phi$, i.e., $\lambda_{i\phi} = \phi_i / (\phi_1 + \phi_2 + \phi_3) / 3$. Although these ratios are calculated based on the initial parameter values of joints, they will be used to calculate the mechanical parameters of the joints of the subareas when $\bar{k}_n$, $\bar{k}_s$, $\bar{C}$ and $\bar{\phi}$ are determined in the orthogonal experiments. For example, if $\bar{k}_n$ is determined, $k_n$ of the joints in the $i$th subarea is $\lambda_{i\alpha} \bar{k}_n$. It should be noted that the average values of the mechanical parameters of the joints listed in Table 3 are different from the experimental factors ($\bar{k}_n$, $\bar{k}_s$, $\bar{C}$ and $\bar{\phi}$). The former are determined from the empirical formula, while the latter are variables that change with the orthogonal experimental scheme. However, the parameter value ranges for the latter are determined by the former.

3.4.2.2. Orthogonal experimental design. Four factors and four levels were selected to perform the orthogonal experiment. For each factor, there are four levels, i.e., I–IV, representing four different values (Table 4). Following the orthogonal experimental method, an orthogonal experimental table was designed (Table 5). In Table 5, the second through fifth columns are the experimental factors. Table 5 lists 16 test schemes; each row represents a test scheme with a level of the four factors. After the level of each factor is set, the value of each factor can be determined according to Table 4. In this manner, the values of the four joint parameters of each rock formation were determined according to the method mentioned in the last paragraph.

3.4.2.3. Computational results of the schemes. Using the joint mechanical parameter values of each scheme listed in Table 5 and computational model shown in Fig. 1, numerical simulation was executed using 3DEC when mining the layer with elevation from $69$ m to $51$ m. The last column of Table 5 lists the $W$-values obtained from the 16 experimental schemes.

3.4.2.4. Range analysis. The changing relation between the indicator and each of the factors is analyzed statistically according to the range analysis method. Taking the factor $\phi$ as an example, the procedure of range analysis is illustrated as follows:

From Table 5, select the schemes in which the values of $\phi$ are set as the value of the I level and calculate the average $W$-value of these schemes. For convenience, this average value is called the Average $W$.

Similarly, calculate the Average $W$ values when $\phi$ is set as the values of the levels II, III and IV, respectively. These Average $W$ values are used as the decision basis for selecting the optimal $\phi$ value.
values describe the relation between the indicator $W$ and $\phi$.

Calculate the difference between the maximum and the minimum of the four Average $W$ values. This difference is named the range value of $W$ associated with $\phi$.

Using the above steps, the changing relation between the test indicator and each of the experimental factors (Fig. 7) and the range values of the $W$ of all the experimental factors were obtained. Note the range values of $\bar{k}_n$, $\bar{k}_s$, $\bar{C}$ and $\bar{\phi}$ are 0.076, 0.443, 0.513 and 1.92, respectively. Obviously, for the $W$, the range values of $\bar{\phi}$ is much greater than those of $\bar{k}_n$, $\bar{k}_s$ and $\bar{C}$, indicating that changes in $\phi$ have greater impacts on the $W$ than changes in other parameters. This range analysis result is close to that of the sensitivity analysis of the joint mechanical parameters from the literature.

3.4.3. Selection of the rock mass mechanical parameters

Using the $W$ of 317 m in the statistical changing relations shown in Fig. 7, the values of the four experimental factors are estimated as $\bar{k}_n = 6.1$ GPa/m, $\bar{k}_s = 2.55$ GPa/m, $\bar{C} = 0.64$ MPa and $\bar{\phi} = 20.8^\circ$.

3.4.4. Validation

We validate the developed computational model and the selected parameters by performing the calculations to simulate the strata movement, and slope failure after the first layer was mined.

3.4.4.1. Strata movement. After completing the mining of the first layer, the surrounding rock-masses of the goaf move and collapse. Fig. 8(a) shows the calculated Z-direction displacements of the rock-masses on the I-I cross section of the model, and Fig. 8(b) shows the calculated Z-direction displacements of rock-masses on the II-II cross section. The positions of section I-I and section II-II can be found in Fig. 2. The following observations can be made from those figures: (1) the roof rock-masses of the goaf collapse downward under the action of gravity, and the ground surface of slope subsides; (2) the traction of the roof collapse causes the wall rock on the north side of the goaf to move toward the mined out space along the dominant joints with occurrence of $220^\circ \pm 20^\circ$. The wall rock on the south side of the mined-out area moves toward the goaf along the dominant joints with the occurrence of $300^\circ \pm 80^\circ$.

3.4.4.2. Movement of ground surface of the eastern pit slope. The collapse and destruction of the wall rocks of the goaf cause slippage and collapse of the slope. To simplify the analysis, we used point “o” in Fig. 6 as the origin to build another cartesian coordinate system. The y-axis is set to the direction that points north at an angle of zero to the horizontal plane, while the x-axis is the direction that points east at an angle of $43^\circ$ to the horizontal plane. The xoy plane of the new coordinate system is nearly parallel to the ground surface of slope above the mined-out area. Fig. 14(a) presents a displacement vector plot for points on the ground surface of slope along the xoy plane. According to the above figures, we obtain the following observations. (1) The ground surface above the goaf slides toward the pit. (2) The ground surface on the north side of mined-out area moves toward the southwest; and the displacement directions are approximately $190^\circ$ to $230^\circ$. (3) The ground surface on the south side of the mined-out area moves toward the northeast; and the displacement directions are approximately $290^\circ$ to $310^\circ$. The above mentioned morphology of the ground surface of the eastern pit slope displacement is generally consistent with the in-situ observations presented in Fig. 6 (a) and (b).
3.4.4.3. Cracks of the ground surface of the eastern pit slope. The DEM can be used to simulate the discontinuous deformation and destruction of rock-masses, but cannot be used to fully describe rock destruction because of the limited number of blocks that can be used in the model. We compensated for this defect using the distribution pattern of the tensile strain on the slope to simulate the cracking damage on the ground surface of slope. The direction of the tensile strain on the ground surface of slope is generally consistent with the direction of the displacement. The impact of the tensile strain causes tensile destruction on the ground surface of slope, and the trending direction of the tensile cracks is nearly perpendicular to the direction of the tensile strain. According to the calculated ground surface of slope displacement direction, the calculated surface crack directions can also be deduced: the directions of cracks on the north side are about 280° to 320°, and the directions of cracks on the south side are about 200° to 220°. We obtained the distribution range of the cracks on the ground surface of slope using an average tensile strain of 5 mm/m as the threshold for the cracking of the slope. This threshold of tensile strain is determined through tri-axial extension tests for the rocks from the three rock formations in the research area. Fig. 6(b) presents a plot of the calculated crack distribution on the ground surface of slope after mining the first layers. The measured distribution range of cracks after mining the first layer on the slope shown in Fig. 6(b), i.e., approximately 317 m, is consistent with the calculated crack distribution range (i.e., approximately 305 m) and the overall crack trending.

The numerically simulated results and the investigated results for the slope displacement, the crack distribution range, and the overall crack trending are in relatively good consistency with each other (Fig. 6(b)). Therefore, it can be claimed that: the developed computational model and the adopted parameters are validated.

3.5. Numerical modeling

As of January 2014, a plan has been followed to exploit the ore bodies on the second to the fourth layers in sequence from the top to the bottom under the eastern final slope, where the elevation ranges for the layers are –87 m to –69 m, –105 m to –87 m, and –123 m to –105 m, respectively (Fig. 4). We simulate the exploitations of those ore bodies on the second to the fourth layer to predict the strata and ground surface movement caused by slice mining.

3.5.1. Strata movement and failure

Figs. 9, 10 and 11 show the movements of the rock-masses on the I-I and II-II cross sections after finishing mining the second, third, and fourth layers, respectively. During the mining process, the patterns of the strata movement and slope failure shown on these figures are as follows:

(1) After the mining of the first layer is completed, the caving of the overburden stratum in the goaf area occurs, and the development of the caving zone reaches the surface (Fig. 8). The collapsed rocks loosely fill the goaf. The rocks in the center of the overburden stratum in the goaf area collapse and are in a loose state; while the rocks in the surrounding slide along the joints but do not collapse. The rocks on the vertical sides of the goaf exhibit relatively small displacements and are basically in a stable state.

(2) After the mining of the second layer is completed, the bottoms of the pillars on the western side of the ore bodies of the first layer (Fig. 4(b)) lose their support, and the roof stratum of the goaf is thus now only supported on three sides (down from four sides previously), resulting in the large-scale caving of the overburdened stratum in the goaf area. Vertical subsidence is the main form of the roof strata displacement. Well-shaped subsidence pits appear on the east final slope. The range of the caving zone at the top of the goaf has expanded, but the roof has not completely collapsed. The rocks on the vertical sides of the goaf have relatively small displacements and remain in a stable state.

(3) After the mining of the third layer is completed, a more than 54-m-tall free surface is formed on each of the side walls of the goaf. Relatively large sliding displacements of the rock-masses on the eastern side wall have taken along the joints with occurrence of 220°±20° and 300°±80°. The rock-masses on the northern and southern side walls slide along the joints with occurrence of 220°±20° and 300°±80°, respectively. The overburden strata of the goaf have been completely destroyed...
and vertically collapsed to fill the goaf.

4) After the mining of the fourth layer is completed, the side walls of the goaf are higher than 64 m. The failure range of each side wall has further increased. Relatively large sliding displacements of the rocks along the joints have occurred. The roof rocks of the goaf have lost their support and subsided completely.

3.5.2. Movement of ground surface of the eastern pit slope

Ore bodies mining results in the movement of the strata, and in turn causes the movement of the east final slope. The maximum subsidence of the ground surface of the eastern slope gradually increases with the increasing mining depth. The maximum subsidence values of the ground surface of the eastern slope are 17 m, 33 m, 51.5 m, and 75.4 m after the mining of the first, second, third and fourth layers is completed, respectively. (Fig. 12). The areas where relatively large subsidence has occurred are mainly concentrated at the top of the goaf, while the other areas of the goaf exhibit a relatively small subsidence. These indicate that vertical subsidence is the main form of the slope surface rock mass displacement during the mining process.

Fig. 13 shows the displacement contours of the ground surface of the eastern slope in the Z-direction after the second to the fourth layers have been exploited. These displacement contours indicate that: as the depth of exploitation increases, the range for the subsidence of the slope surface significantly expands.

Fig. 14 shows vector plots for the displacement of points on the slope ground surface along the xoy plane after the layers have
been exploited. The direction of the displacement of individual regions on the ground surface of slope essentially remains constant after the layers have been exploited; and the ground surface above the goaf primarily slides to the open pit. The slope on the north side of the mined-out area moves southwest, and the displacement directions are approximately 190° to 225°. The slope on the south side of the mined-out area moves west and northwest, while the displacement directions are approximately 245° to 290°. The values of slope ground surface movement increase along with the increasing of the mining depth.

3.5.3. Cracks of ground surface on the eastern pit slope

We used a tensile strain of 5 mm/m, which is determined according to tension tests of rock samples from the Yanqianshan iron mine, as the threshold of ground surface cracking to plot the calculated crack distribution on the ground surface after each layer has been exploited (Fig. 15). It can be also observed that: the widths of the crack distribution range are 379 m, 388 m and 397 m after finishing mining the second, third and fourth layer, respectively.

![Graph showing ground surface subsidence of the eastern slope along the II-II in Fig. 2.](image)

![Diagram of displacement contours of the ground surface on the eastern final slope in the Z-direction after completing the mining of the: (a) second layer; (b) third layer; and (c) fourth layer.](image)
4. Discussions and conclusions

4.1. Discussions

Mining under the final slopes of an open pit induces the mechanical behaviors such as caving and large displacement movements in the overburden strata and the surrounding rocks of the goaf. How to reasonably simulate the mechanical behaviors of rock-masses is a difficult issue in the numerical simulation for underground mining. The employment of the Equivalent Jointed

---

**Fig. 14.** Vector plots for the displacement of points on the slope ground surface along the xoy plane after completing the mining of the: (a) first layer; (b) second layer; (c) third layer; and (d) fourth layer.
Rock-mass Model-based 3D DEM has the capability to overcome this difficulty. In the present study, we add several sets of equivalent joints to the 3D geological model of the study object, cut the model into blocks of a suitable size, and thus form an EJRM. This model can not only be suitable for simulating the caving and movement of the rock-masses in the overburden strata and the surrounding rocks, but also ensure that the movement of the strata is controlled by the joints added. Furthermore, the moderate number of joints in the model ensures that the calculation can be performed smoothly.

The EJRM is formed through simplification based on the actual rock mass structure of the study object and inherits the basic characteristics of the actual rock mass structure, including the stratigraphic structure, the terrain and the distribution of dominant joint sets, etc. However, due to the limitation of the calculation efficiency, we cannot consider too many joints in the model. Therefore, the EJRM is different from an actual rock mass structure model. An approach to compensating for this defect is as follows: the mechanical parameters of the EJRM are determined based on back analysis, and then numerical simulation verification is conducted to allow the calculation results of EJRM-based numerical simulation to be consistent with the measured data. The results of verification through numerical simulation conducted in the present study are summarized herein. (1) The strike directions of the tensile cracks located on the east final slope on the northern and southern sides of the ore bodies are calculated to be about 280–320° and 200–220°, respectively, which are effectively close to the measured strike directions of the slope cracks. In addition, these tensile cracks also correspond to the dominant joint sets on the northern and southern sides with an occurrence of 220°±20° and 300°±30°, respectively, indicating that the slope surfaces on the northern and southern sides primarily slide along the two joint sets. (2) After the mining of the first layer is completed, the distribution width of cracks is calculated to be 305 m, and is very close to the measured distribution width of 317 m. The verification demonstrates that the calculation results approximate to the measured ones; thus, the equivalent jointed rock mass model established in the present study and its parameters given are suitable.

Because of a huge goaf area and violent mining-induced disturbance, mining under the open pit will result in discontinuous, large displacement motions and failure of the strata and slopes. The analysis of the example used in the present study demonstrates that the maximum subsidence of the overburden strata reaches up to 72.8 m and that the roof strata of the goaf vertically subsides during the mining process, which eventually forms loose deposits. This phenomenon is consistent with reality. It is difficult for continuous medium mechanics-based FEM and FDM to simulate the discontinuous, large displacement motion failure mode of rock-masses, which is the reason why the 3D DEM was selected in the present study.

4.2. Conclusions

An EJRM-based numerical modeling method is presented in this paper for modeling the strata and surface movement caused by mining under an open pit final slope. This method involves three main procedures: (1) building an equivalent jointed rock mass model; (2) inverting mechanical parameters of the model by using back analysis; and (3) modeling the underground mining to predict the laws of mining induced strata and surface movement and failure.

To demonstrate the effectiveness of the proposed method, it is applied to simulate the underground mining in the Yangqianshan iron mine. It has been concluded that: (1) the displacement and destruction of surrounding rocks in the goaf area is mainly controlled by (a) the gravity of rocks and (b) the characteristics of the joint distribution; (2) the traction of the movement and destruction of surrounding rocks of the goaf area leads the slope surface to subside downward and slide inward to the pit; and (3) vertical subsidence is the main form of the roof strata displacement and a well-shaped subsidence pit is forming on the east final slope as the mining depth increases.

The EJRM-based numerical modeling method can not only simulate the complex mechanical behavior of rock-masses, such as caving and large displacement movement, but also overcome the computational difficulty caused by the use of a large number of blocks in the calculation model. However, the most important shortcoming of our method is the troublesome characterization of sliding along discontinuities. This problem is probably due to the insufficient iteration steps in the numerical calculation. Future efforts are planned to be carried out to deal with this problem.

Acknowledgments

This research was supported by the Natural Science Foundation of China (Grant no. 40602037, 40872183, and 51541405), China Postdoctoral Science Foundation (2015M571081), and the Fundamental Research Funds for the Central Universities (2652015065, 2652015319). The authors would like to thank the editor and the reviewers for their contributions on the paper.

References


