Full-wave-equation depth extrapolation for true amplitude migration based on a dual-sensor seismic acquisition system

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SUMMARY
Most depth extrapolation schemes are based on a one-way wave equation, which possesses limited ability to provide the true amplitude values of reflectors that are highly important for amplitude-versus-offset inversion. After analysing the weaknesses of current migration methods and explaining the reason why wavefields cannot be extrapolated using the full-wave equation in the depth direction, a full-wave-equation migration method based on a new seismic acquisition system is proposed to provide accurately dynamic information of reflection interfaces for migration. In this new seismic acquisition system, double sensor data are provided to solve the acoustic wave equation in the depth domain accurately. To test the performance of recovering the true amplitudes of the full-wave-equation migration, we used a single shot gather and several multiple shot gathers produced by a 2-D numerical modelling technique to demonstrate that our methodology provides better estimated true amplitudes than that of the conventional Kirchhoff and reverse time migration algorithms through comparison of the amplitudes of the target reflectors with its theoretical reflection coefficients. Because double sensors are applied to implement the full-wave-equation migration, it is necessary to study the perfect distance between the double sensors to diminish the migration error for future practical exploration. Based on the application of the full-wave-equation migration method to the first set of actual seismic data collected from our double sensor acquisition system, our proposed method yields higher imaging quality than that of conventional methods. Numerical experiments and actual seismic data show that our proposed method has built a new bridge between true amplitude common-shot migration and full-wave-equation depth extrapolation.

Key words: Image processing; Numerical solutions; Non-linear differential equations; Wave propagation.

INTRODUCTION
In recent decades, one-way wave equations for seismic migration, originally proposed by Claerbout (1971, 1985), have been widely used in seismic exploration and dominate the finite-difference migration methods. From the 15 degree equation to the Fourier finite-difference algorithm (Ristow & Rühl 1994), one-way wave equation finite-difference migration algorithms have experienced a great development and can not only provide increasing accurate traveltime information of complex structures but also image complex structures with large lateral velocity variations effectively and efficiently. However, the standard one-way wave equation finite-difference methods are unable to provide accurate amplitudes because we have to split the full-wave equation into its upward- and downward-propagating parts for depth continuing operations (Zhang et al. 2003, 2005). The usage of one-way wave equations leads to a correct result only in the homogeneous media but not in the heterogeneous media. The reason why we can only use the downgoing propagator to compute extrapolations is that the spatial derivative value of the wavefields cannot be recorded, so that an initial value problem with respect to a spatial variable for a second-order wave equation is unsolvable using numerical approaches (Sandberg & Beylkin 2009). Therefore, it seems to be arduous by nature to produce true amplitudes for reflectors using the conventional one-way wave equation migration algorithms (Kosloff & Baysal 1983). The original vision of Kirchhoff integral solution about Huygen’s principle can be written as below (Shearer 2009):

$$u(x, y, z, t) = \frac{1}{4\pi} \iint_{\bar{V}} \left\{ \frac{1}{r} \frac{\partial u}{\partial n} - \frac{\partial}{\partial n} \left( \frac{1}{r} \right) [u] + \frac{1}{\nu} \frac{\partial}{\partial r} \left[ \frac{\partial u}{\partial r} \right] \right\} dQ. \tag{1}$$

Because the present seismic acquisition system cannot observe the spatial derivative of wavefields, researchers have proposed various ways to address the second term in eq. (1). The aim of those methods is to apply some assumptions to discard the second term of eq. (1) in a fitting manner. These approximations convert...
eq. (1) into a practical migration formula that precisely and effectively calculates stratigraphic images of the subsurface region but does not succeed in recovering the reflectivity. The strength of reverse time migration relies upon the fact that it uses a two-way acoustic wave equation for both the forward and reverse time extrapolations because the initial value problem of the full-wave equation can be solved in time instead of that in depth, thus improving imaging in areas with complex geology and overcoming the assumptions made in the Kirchhoff or one-way wave equation migration methods. There are three weaknesses of reverse time migration: the high computational memory cost, the low efficiency of the migration in actual seismic data processing and the existence of low-frequency artefacts that affect the estimation of the true amplitudes for reflectors (this part will be discussed in the latter paragraph of this paper). Additionally, when we resolve the wave equation in the time direction, wavefields are assumed to be null at the time beyond the maximum of the resampling time (Baysal et al. 1983). Because of this hypothesis, the computed amplitudes of the reflectors have a gap from the theoretical reflectivity.

Currently, industrial users are not satisfied with only the stratigraphic images given by the conventional one-way migration or Kirchhoff migration methods but have increasingly requirements for true amplitude migration results. In addition, the seismic inversion of the amplitude variation with offset (AVO) or amplitude variation with angle for reservoir description is a high-level requirement for amplitudes of seismic migration (Deng & McMechan 2007). Many researchers have found that migration prior to AVO can produce more accurate and reliable seismic migration amplitudes (Resnick et al. 1987; de Bruin et al. 1990; Beydoun et al. 1994). Much research has tried to produce accurate amplitudes using finite-difference migration methods. Zhang et al. (2003, 2005) proposed a modified one-way wave equation migration called ‘true amplitude wave equation migration (WEM)’, which overcomes the 90 degree limitation of the propagation operator and can strengthen the image of large dip structures by introducing a true amplitude correction term. Mittet et al. (1995) and Causse & Ursin (2000) proposed viscoscalar and viscoacoustic finite-difference extrapolations, respectively, which are still based on a one-way wave equation. The weighted Kirchhoff migration methods carried out by several authors attempt to obtain the true amplitude migration, but they are still based on approximate Kirchhoff equations (Bleistein 1987; Berkhout & Wapenaar 1993; Hanitzsch et al. 1994). Deng & McMechan (2007) presented a new pre-stack depth-migration scheme that uses the framework of reverse time migration to compensate for geometric spreading, intrinsic $Q$ losses and transmission losses.

The root of the entire problem with current migration methods is the seismic acquisition system, which can only collect wavefields on the surface but miss the spatial derivative of the wavefields at the surface. We therefore have to split the full equation in finite-difference migration methods and abandon the spatial derivative of the wavefields in the original Kirchhoff migration schemes, which affect the amplitudes of the reflectors to a certain extent. The target of our study is to tackle the problem of finding a migration scheme that can perform depth extrapolation based on the full-wave equation, which some research papers have focused on. To fulfil migration by the full acoustic wave equation in the depth domain, two boundary conditions should be known to initialize the computation of the depth extrapolation. Kosloff & Baysal (1983) and Sandberg & Beylkin (2009) generated the normal derivative of the wave field at the surface from a mathematical assumption. This assumption is only suitable for the situation in which the velocity is laterally uniform on the surface. Aiming to move beyond this assumption, we attempt to design a new type of seismic acquisition system to provide us with the information we need: a normal derivative of the wavefields at the surface.

In this paper, in combination with the new seismic acquisition system, we propose a mathematical formalism and an algorithm for depth extrapolation that we call full-wave-equation migration (FWEM) based on double sensors. This paper lists some synthetic examples to study the performance of recovering true amplitude information. The common shot synthetic gathers were calculated using a high-order staggered grid finite-difference scheme, and the perfect match layer absorbing boundary condition is used to eliminate the reflections from the model boundary in those examples. To test whether our proposed method is available for use in actual seismic exploration, a set of seismic data is collected, and the imaging differences between the conventional migration and the FWEM are shown.

**NEW SEISMIC ACQUISITION SYSTEM**

In the new updated system, there are two seismic detectors located at one station, where one is on the surface $Z_0$ and the other is just below, at a certain depth $Z_1$, as per the simplified diagram shown in Fig. 1.

A source $O$ provides a seismic signal that is reflected by an interface and received by receivers (e.g. $R_{0i}$ and $R_{1i}$). Although the reflected waves received by detectors $R_{0i}$ and $R_{1i}$ might not come from the same reflected points, such as $r_1$ and $r_1$, generally speaking, the distance between the surface and the reflector is generally much larger than the distance between the two sensors. Hence, it is reasonable to reckon that the reflected waves received by the up/down detectors are from the same point.

![Figure 1. The dual-sensor acquisition system.](http://gji.oxfordjournals.org/)

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FULL-WAVE-EQUATION DEPTH MIGRATION

In a 2-D medium, the initial-value problem of the acoustic wave equation based on two sensor data is given as follows:

\[
\begin{align}
\frac{\partial^2 u}{\partial t^2} + \frac{\partial^2 u}{\partial z^2} &= \frac{1}{v^2} \frac{\partial^2 u}{\partial t^2} \\
u(x, z = 0, t) &= d(x, 0, t) \\
\frac{\partial}{\partial z} u(x, z = 0, t) &= \frac{d(x, \Delta z, t) - d(x, 0, t)}{\Delta z}
\end{align}
\]  

(2a) \hspace{1cm} (2b) \hspace{1cm} (2c)

Where \(x\) and \(z\) are the horizontal and vertical coordinates, \(u(x, z, t)\) is the wavefields at time \(t\), \(v(x, z)\) is the velocity of the medium and \(\Delta z\) is the grid space and the continuing step. Equation (2c) means that two sensors that recorded seismic data \(d(x, 0, t)\) and \(d(x, \Delta z, t)\) at \(z = 0\) and \(z = \Delta z\), respectively, are used to compute the normal derivative \(u_x(x, z, t)\) at \(z = 0\), which makes the wave equation exactly solvable.

However, it is difficult to use eq. (2b) to extrapolate wavefields in the space-time domain. The most feasible method is to solve eq. (2) in the frequency domain. Then, we apply the Fourier transform in the time direction to eq. (2a), and the result is shown as below:

\[
\frac{\partial^2 \tilde{u}(x, z, \omega)}{\partial z^2} = \left[ -\frac{2\pi \omega}{v(x, z)} - \frac{\partial^2}{\partial x^2} \right] \tilde{u}(x, z, \omega) = L \tilde{u}(x, z, \omega)
\]

(3)

where \(\omega\) is the temporal frequency and \(\tilde{u}(x, z, \omega)\) presents transformed wavefields.

Considering eq. (3) with the initial conditions (2b) and (2c), the full-wave-equation depth migration scheme can be written as a first-order system

\[
\begin{bmatrix}
\frac{d}{dz} \tilde{u} \\
\frac{d}{dz} \tilde{u}_x
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
L & 0
\end{bmatrix}
\begin{bmatrix}
\tilde{u} \\
\tilde{u}_x
\end{bmatrix}
\]

(4)

where \(\tilde{u}(x, z = 0, \omega)\), \(\tilde{u}_x(x, z = 0, \omega)\), \(\hat{d}(x, 0, \omega)\) and \(\hat{d}(x, \Delta z, \omega)\) are the Fourier transform results of \(u(x, z, t)\), \(u_x(x, z, t)\), \(d(x, 0, t)\) and \(d(x, \Delta z, t)\) with respect to time respectively (Mavko et al. 2003).

To solve eq. (4) for the downward continuation within the frequency domain, it is necessary to design a filter to remove evanescent waves (Sandberg & Beylkin 2009) that can cause the numerical solution to increase exponentially in operator \(L\) during depth extrapolation. Two methods are suggested to address this problem: the ideal low-pass cut-off filter described by Kosloff & Baysal (1983) and the spectral projector method given in Sandberg & Beylkin (2009), who proposed the application of a spectral projector \(P\) to the unstable operator \(L\) to create a stabilized operator \(PLP\). The spectral projector \(P\) is defined as (Kenney & Laub 1995):

\[
P = (I - \text{sign}(L))/2.
\]

Here, \(I\) means the identity matrix and \(\text{sign}(L)\) is the matrix sign function of the matrix \(L\). We provide a comparison of those two filters later in this paper, finding that the ideal low-pass cut-off filter not only affects the imaging correctness of steep structures, as discussed in Sandberg & Beylkin (2009) but also cuts off some useful frequency components of the normal propagating wavefields in a large lateral velocity variation at a given depth, resulting in failing to obtain reliable true amplitude information. Hence, the spectral projector method is studied well in our work.

In our research, the operator \(L\) in eq. (4) need be converted to a matrix form of sampling in space. In our numerical experiment, a 10th-order numerical discretization is considered about \(D_x\) to achieve high accuracy and suppress dispersion as much as possible. Then, the spectral projector algorithm that was used in Sandberg & Beylkin (2009) and Miao et al. (2014) is adopted to solve the evanescent wave problem. The biggest challenge of the spectral projector method is that we have to calculate the spectral projector of every single frequency at a given depth, which is infeasible if the band of frequency is wide. This is also the reason that previous researchers had to use a low frequency, such as 7 or 10 Hz of dominating frequency. In their research (Sandberg & Beylkin 2009; Miao et al. 2014), some measurements achieved a desirable acceleration ratio but not enough. To address the computing expense in terms of speed, the graphic processing unit accelerating technique is added to the algorithm to speed up the depth continuation.

When we look back to the derivation of the migration algorithm, there is no assumption in it. Although the propagation of wavefields is irreversible in physics, the process can be reversed in mathematics. Our proposed FWEM method unconditionally abides by the acoustic wave equation. Consequently, our proposed method is able to image all types of waves and theoretically provide true amplitudes of the reflectors.

At present, reverse time migration is the only FWEM method and it performs migration in the time direction (Baysal et al. 1983). Reverse time migration is currently the best migration method because it can clearly and accurately image various types of waves and complex structures. As our proposed FWEM and reverse time migration are both based on two-way propagators, it is necessary to analyse and compare the performance in recovering the true amplitudes of both methods.

To obtain accurate image amplitudes, with the caveat that the full-wave-equation must be applied for extrapolation, pre-stack depth migration also requires correct implementation of the imaging condition. After evaluating the reflection coefficients estimated by six imaging conditions for pre-stack reverse-time migration (Chattopadhyay & McMechan 2008), the deconvolutional imaging condition has been used to preserve the physical meaning of the imaging amplitude. It is unavoidable to produce the low-frequent artefacts by using the deconvolutional imaging condition in the FWEM, hence it is necessary to apply a derivative of the image in the vertical dimension to remove that.

Synthetic example test and discussion

The goal of this section is to show the performance of our scheme in recovering true amplitudes in various synthetic examples. Synthetic common shot gathers are generated using a high-order staggered grid finite-difference technique based on a 2-D acoustic wave equation. All the examples involve necessary geometric spreading compensation. We used three migration methods to compute the migration section with multiple shot examples: the first method is the conventional Kirchhoff pre-stack depth migration; the second is our proposed full-wave-equation depth migration; and the third is reverse time migration. For evaluation and comparison, reflection coefficients are estimated for the target reflectors in the migration sections and compared with the theoretical reflection coefficient values. The theoretical reflection coefficients are calculated by the Aki–Richard equation (Aki & Richards 1980) in the single shot experiment.
is used in the forward calculation. The first receivers are placed at $z = 100 \ldots 600$, where $\Delta z = 1$ m and $\Delta x = 1$ m are grid spacing of velocity model. Single shot records of double sensors after geometric spreading compensation are used to implement the depth extrapolation. Fig. 2(b) is the migration result after our proposed FWEM. The comparison between the theoretical reflection coefficients and the estimated amplitude values along the target reflector from the seismic section in the horizontal range from 300 m to 700 m is given in Fig. 2(c).

From this single shot example, the estimated amplitude values along the target reflector using our proposed method fit well with the theoretical values, even at a large incident angle, which preliminarily demonstrates the true amplitude character of our proposed FWEM. Because a lack of actual double sensor data, we used the finite-difference technique to simulate the actual multiple seismic coverage observation system to compute multiple shot gather in several complex unconformity models that are constructed to understand the quality of the calculated amplitudes of the target reflectors. We built unconformity models to verify whether the migration methods we used could also distinguish lithological differences.

**Dipping unconformity model**

To further test whether our proposed method is able to recover true amplitudes for a large dipping angle model with multiple shot gather, a 45 degree dipping model with an unconformity interface is created. The velocity model is presented in Fig. 3(a). In this model, A Ricker pulse with a dominating frequency of 60 Hz is employed in the forward modelling. The sources at shot locations are placed at $x_i = i + 3 \Delta x$, $i = 0 \ldots 263$, with $\Delta x = 1$ m. For each source, the first receivers are placed at $x'_i = x_i + j \Delta x$, and the second receivers are placed at $x''_i = x_i + j \Delta x + \Delta z$, $j = 0 \ldots 59$, with $\Delta z = 1$ m. Figs 3(b)–(d) show the migration sections of the conventional Kirchhoff pre-stack depth migration, our proposed FWEM and reverse time migration about this model, respectively. Due to limitations caused by the horizontal length of the model, the reflection waves from deep positions of the dipping model are generally not recorded, so that deep reflection events are lost in the migration section. The selected amplitude curves comparisons with the theoretical reflection coefficients of the three results are drawn in Figs 3(d), (e) and (g). For the same reason, in Figs 3(d), (e) and (g), we only analyse the helpful parts of the selected amplitudes. Although the dipping angle of the reflector is slightly large, the estimated amplitudes of our proposed method are still generally identical with the theoretical reflection coefficients, while the amplitudes obtained from the conventional Kirchhoff and reverse time migration methods deviate lightly from the theoretical values. Therefore, our method inherently has a better ability to recover the true amplitude even for the large dip angle layer.

According to the numerical results of this model, our method not only offers the correct depth positions of target reflectors but also has the potential to divide specific lithological sequences and has the capacity to provide more accurate and reliable true amplitude information than that obtained from the conventional Kirchhoff and reverse time migration methods.

**Model with two interfaces**

Generally speaking, it is not sufficient to only analyse the reflection coefficients of a single layer. For the third model, there are two interfaces to discuss whether a dipping interface would influence the

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**Figure 2.** (a) The dipping layer velocity model; (b) migration result of our proposed full-wave-equation migration; (c) comparison of the reflection coefficients estimated along the target reflector in the dipping layer model with the theoretical values.
Figure 3. (a) The velocity model of the 45 degree dipping unconformity; (b) migration section after the conventional Kirchhoff pre-stack depth migration method; (c) migration section after our proposed full-wave-equation migration method; (d) migration section after reverse time migration method; (e) comparison of picked amplitude obtained from the dipping layer image in panel (b) with theoretical reflection coefficient; (f) comparison of picked amplitude obtained from the dipping layer image in panel (c) with theoretical reflection coefficient; (g) comparison of picked amplitude obtained from the dipping layer image in panel (d) with theoretical reflection coefficient.
imaging energy on the second interface by using different migration methods and also to study the overall reflection coefficients of the model; the velocity model is shown in Fig. 4(a). The velocity of the upper layer of the dipping interface is 2000 m s\(^{-1}\), and that of the layer below the dipping interface is 3500 m s\(^{-1}\), and the dipping angle is 30 degrees, so the lateral velocity difference of the dipping interface in velocity is 1500 m s\(^{-1}\). The velocity in the orange-coloured zones is 4000 m s\(^{-1}\), and that in the red zone is 4500 m s\(^{-1}\). A Ricker pulse with a dominating frequency of 60 Hz is used in the forward calculation. The sources at shot locations are placed at \(x_i = i \times 6 \Delta x, i = 0, \ldots, 113\), with \(\Delta x = 1\) m. For each source, the first receivers are placed at \(x_{i,j} = x_i + j \Delta x\), and the second receivers are placed at \(x_{i,j} = x_i + j \Delta x + \Delta z, j = 0, \ldots, 119\), with \(\Delta z = 1\) m. The three migration sections that we calculated by the three methods are shown in Figs 4(b)–(d). Then, we estimate the coefficients about the dipping interface and the unconformity interface, and the estimated and theoretical coefficient values are drawn in the same picture for comparison in Figs 4(e)–(g).

Because of the different continuing programs we used, it is difficult to directly match the selected amplitudes of the reflectors with the reflection coefficients, and they may not even reach an approximate order of magnitude. Therefore, the overall migration section is scaled by a constant factor to reach the same amplitude range as reflectivity, with the relative energy relations of the different imaging reflectors maintained unchanged. An integrated analysis of Figs 4(e)–(g) clearly illustrates that there is a relatively large bias between the reflecting amplitudes after the conventional Kirchhoff migration and reverse time migration approaches compared with the theoretical coefficients, while our proposed FWEM algorithm leads to a relatively high imaging quality of recovering the true amplitudes, except for the boundary region. Our proposed method can handle the kinematic aspects of the imaging problem but also includes the dynamic information, which is increasingly important for studies of amplitude variation with the offset and angle. Using the conventional Kirchhoff migration algorithm, it is very difficult to preserve the amplitudes of the reflectors in the deep and shallow zones of the model and make both of them match well with the theoretical coefficients, although it can clearly address the positions of the reflectors. Interestingly, for reverse time migration, although the selected amplitudes of the second interface match well with its reflection coefficients, the selected amplitudes of the first interface produce a certain amount of bias from its theoretical reflection coefficients. This can be interpreted that when a wave passes through the first interface, a massive amount of energy is lost, called transmission losses, so that the imaging energy of the second interface is low with only geometric spreading compensation. The imaging energy of the second interface can be compensated by introducing other compensating forms such as transmission compensation.

In Fig. 4(h), the selected amplitudes of the two reflectors are estimated using the FWEM with the ideal cut-off filter proposed in Kosloff & Baysal (1983). Through a comparison of the migrated amplitudes in Figs 4(h) and (f), we note that the selected amplitudes of the reflectors represent a subtle gap from the theoretical values. This means that the ideal cut-off filter removes not only the evanescent waves but also some useful parts of waves, which will lower the imaging quality of the true amplitudes.

### Model with multiple interfaces

To investigate whether our proposed FWEM method remains valid for multiple interfaces, a complex model is generated in this example, as shown in Fig. 5(a). The synthetic shot gathers are calculated using a Ricker pulse with a dominating frequency of 60 Hz. The sources at shot locations are placed at \(x_i = i \times 4 \Delta x, i = 0, \ldots, 120\), with \(\Delta x = 4\) m. For each source, the first receivers are placed at \(x_{i,j} = x_i + j \Delta x\), and the second receivers are placed at \(x_{i,j} = x_i + j \Delta x + \Delta z, j = 0, \ldots, 119\), with \(\Delta z = 4\) m. To compare the performance of three migration methods in computing the true amplitudes, those migration sections are put together (Fig. 5), and the amplitudes of the three target reflectors (H1, H2 and H3) are estimated from their respective imaging sections. Then, the comparison of the amplitudes calculated about the target reflectors with its corresponding theoretical coefficients are presented in Figs 6–8. To quantitatively analyse the differences between the calculated amplitudes and theoretical coefficients, the relative error is given using the formula

\[
E_{\text{relative}} = \frac{A_{\text{theoretical}} - A_{\text{calculated}}}{A_{\text{theoretical}}} \times 100\%.
\] (5)

The relative errors of the reflection amplitudes generated by the three migration methods about the reflectors are pictured in Figs 6–8.

Viewing the migration sections produced by the three migration methods, it is clear that there is no difference in structure imaging between the three migration methods. All of them can image the exact positions of the reflectors. The main noteworthy disparity occurs in calculating the reflection coefficients of the target reflectors. To appropriately compare the calculated reflection coefficients with the theoretical coefficients, each migration section is multiplied by a respective constant value. After contrasting the calculated reflection coefficients with the theoretical coefficients about the target reflectors in the three migration sections and analysing their relative errors, it is clear that our proposed FWEM method can provide more credible reflection coefficients with minimum controllable errors even for multiple interfaces and the overall trend errors are no more than 20 per cent except the intersections of different interfaces, while the conventional Kirchhoff pre-stack depth migration method and reverse time migration method produce relatively higher errors from the theoretical reflection coefficients. This example gives us a proof of the performance of true amplitude migration in our proposed approach, that it can accurately distinguish lithological changes along the reflectors. The reason why the reflection coefficient value of reflector H2 jumps up and down is that its interface is a ladder-like using regular grid subdivision.

### Influence of distance changes of double sensors on imaging quality

In this paper, we explained that the seismic data of double sensors are used to precisely solve the second-order spatial derivative of the wave equation. In the above studies, we set the distance of the double sensors equal to the extrapolation step and the grid size of the forward modelling. In this section, we want to analyse the influence of changing in the distance of the double sensors has on the migration result. The results will be conducive to designing a receiver device containing the function of providing a reliable normal derivative of wavefields with a minimum design cost for future practicable applications.

To study the influence, we set the distance of the double sensors as 0.1, 0.3, 0.5, 0.7 and 1.0 m to record the seismic reflection waves. The grid size of the other parts of the model is still 1.0 m. A 30-degree dipping model with an unconformity interface is constructed; its velocity model is shown in Fig. 9(a). To collect the wave field at
Figure 4. (a) The velocity of a two-interface model; (b) migration section of the conventional Kirchhoff pre-stack time migration method; (c) migration section after our proposed full-wave-equation migration method; (d) migration section after reverse time migration method; (e) comparison of picked amplitude obtained from a two-layer image in panel (b) with theoretical reflection coefficient; (f) comparison of picked amplitude obtained from the two-layer image in panel (c) with theoretical reflection coefficient; (g) comparison of picked amplitude obtained from the two-layer image in panel (d) with theoretical reflection coefficient; (h) comparison of picked amplitude obtained from the two-layer imaging using the ideal cut-off filter in full-wave-equation migration method with theoretical reflection coefficient.
A new true amplitude migration method

Figure 5. (a) The velocity model; (b) migration section after the conventional Kirchhoff pre-stack depth migration method; (c) migration section after our proposed full-wave-equation migration method.

$z = 0.1$ m, the regular conventional finite-difference schemes are obviously disabled to meet the requirement. Hence, the spatial variable grid finite-difference technique is introduced, and its configuration is plotted in Fig. 9(b). A high-order finite-difference method with a variable grid is employed to simulate the seismic responses to avoid causing any significant artificial noise in the transition zone between the fine and coarse grids (Tessmer 2000; Huang & Dong 2009).

Five curves that are picked from a target reflector in the migration sections and a theoretical reflection coefficient curve and the relative errors between them are illustrated in Fig. 10. After comparing these curves, we find that the computed reflection coefficients as the distances of the two sensors are 0.1, 0.3 and 1.0 m that matches best with the theoretical reflection coefficient curves, while the estimated reflection coefficients jump up and down at distances of 0.5 and 0.7 m, indicating great instability and presenting large fluctuations. We analyse the usage of a forward differential equation to predict the diveritive data at the $z = 0$ position. There is an error between the predicted seismic data and the true recording seismic data. The forward differential equation produces unacceptable errors for computing true amplitudes as the distance of the dual sensors increases. We thought that the best designed distance of the double sensors is the extrapolation step, so that the secondary sensor can record the true reflection wavefields. Yet, it is difficult to meet this requirement in practical seismic exploration. Technically, we observed that when the distance is sufficiently short (such as 0.1 or 0.3 m), it can achieve the same estimated amplitudes as the result using the continuing step as the distance of the double sensors. We therefore advise that using the distance is about 10–30 per cent of the continuing step in the design of a double sensor seismic acquisition system can best image the subsurface structures efficiently.

Application of actual seismic data

Whether our proposed FWEM method can obtain satisfactory results in actual situations is our greatest concern. To answer this question, we specially designed a double-sensors acquisition system and collected the first set of double sensor seismic data. Because we do not have this type of device to record the derivative of the wavefields at present, we have to use two geophones to gather seismic data: the first on the surface, and the other at the bottom of a 0.5 m deep hole. The forwarding difference equation is a good choice to compute the derivative of the wavefields about the depth direction. Considering the efficiency and ease of operation, we fixed the receiver array and moved the shot positions to perform multiple coverage data acquisition. Conventional processing methods are used to pre-process the dual sensor data. Then, the conventional Kirchhoff migration method and our proposed migration method are utilized to image the subsurface structures, as shown in Fig. 11.

From the comparison of Fig. 11, it is understandable that the use of the full-wave-equation method as a true amplitude migration algorithm improved the imaging quality. As we operate the subtraction between the dual sensors, it can suppress part of noise and improve the signal-to-noise ratio of the migration section. The best visible improvements are obtained in the areas marked by the red boxes. Observe that in these areas, the image obtained from our FWEM is clearer than the corresponding areas in the conventional Kirchhoff image migration section. It is clear that the
contact relations between events are more distinct in our FWEM section; so that we can more directly interpret the non-conformable contact, which is of great importance for AVO inversion, seismic attribution studies, and other applications. The innate features of the FWEM algorithm make the accurate imaging of varied waves possible.

**Further Discussion**

Here, we present a deeper discussion of reverse time migration and our proposed FWEM algorithm. Why these two methods are combined for the purpose of comparison is that they are based on the full-wave equation. Because the conventional seismic acquisition system can only provide the wavefields on the surface, solving the
A new true amplitude migration method

Figure 7. (a) Comparison of picked amplitudes obtained from the reflector H1 in migration section imaged by our proposed full-wave-equation migration method with theoretical reflection coefficients; (b) the relative errors between calculated reflection coefficients and theoretical reflection coefficients about the reflector H1; (c) comparison of picked amplitudes obtained from the reflector H2 in migration section imaged by our proposed full-wave-equation migration method with theoretical reflection coefficients; (d) the relative errors between calculated reflection coefficients and theoretical reflection coefficients about the reflector H2; (e) comparison of picked amplitudes obtained from the reflector H3 in migration section imaged by our proposed full-wave-equation migration method with theoretical reflection coefficients; (f) the relative errors between calculated reflection coefficients and theoretical reflection coefficients about the reflector H3.

full-wave equation in the depth domain will trigger an instability issue. Hence, reverse time migration rotates the Cartesian coordinates from the depth direction to the time direction and assumes that the wavefields are zeros when the sampling time exceeds the maximum recorded time. The disadvantage of this assumption we have already discussed. The use of the conventional cross-correlation imaging condition in reverse time migration will inevitably produce low frequency artefacts, which often blur the geological structures. Our proposed method is based on the downward continued survey, in which we downwardly continue all source and receiver gathers from $z_n$ to $z_{n+1}$, resulting in a new survey. When the conventional cross-correlation imaging condition is applied in this latest survey,
we do not need to image the area above the new survey so that it will generate less low-frequency artefacts.

In addition to this, our proposed full-wave-equation migration algorithm is more easy to manipulate than reverse time migration. In the standard reverse time migration, the forward-propagating wavefields must be visible to the downward-propagating wavefields, and vice versa. That means we have to at least store wavefields of one propagating direction at every sampling time. To view this question from the depth aspect, every shot gather at per depth needs to be memorized in reverse time migration, while just two shot gathers are required in our proposed full-wave-equation depth migration, as well as generating more credible amplitude values. Because of this
A new true amplitude migration method

Figure 9. (a) The velocity model of a 30 degree dipping unconformity; (b) variable spatial grid configuration.

great superiority, our two-way depth migration requires less computer memory than the standard reverse time migration, therefore it has great practical significance in actual seismic processing. The dual sensor acquisition system can also provide extra seismic data so that it is helpful for us to suppress noise and possibly eliminate or utilize multiple waves efficiently.

Comparing the images from our proposed full-wave-equation depth extrapolation and the standard reverse time migration, it is quite obvious to notice that if an additional measurements can be obtained like the one we did in the our migration scheme, It is quite possible to get an improvement in image for other imaging techniques (such as Gaussian beams depth extrapolation) that can be adapted to deal with the dual sensor data (El Yadari 2015). Recently, some researchers have attempted to combine the multimeasurement data from dual marine (or OBC) sensors in reverse-time migration (Vasconcelos 2013; Ravasi et al. 2015). Given their research, including the dual sensor data in vector reverse time migration is achievable based on the relation between the measurements of the particle velocity vector and the spatial gradient of pressure in the acoustic media. The spatial gradient of pressure can also be estimated by the measurements of the particle velocity vector in the multimeasurement data of dual streamer or OBC sensors, therefore it is feasible to incorporate the information from dual marine (or OBC) sensors in the full-wave-equation depth migration. This two research fields are worth digging deeper in our further research.

CONCLUSIONS

In this paper, we explain that we use a one-way wave equation to perform depth extrapolation because the spatial variable for a second-order wave equation is ill-posed and its solution unstable by using only one initial value condition. We know that the one-way wave equation migration method is unable to correctly estimate the true amplitude. Although the current best migration method, reverse time migration, can provide fairly correct estimated amplitudes of reflectors, its high hardware requirements are the greatest obstacle to its application in practical explorations. The target of this paper is to propose a new method that can calculate the true amplitude of reflectors with high efficiency by designing a new seismic acquisition system. To complete the FWEM without any assumptions on the wave propagation, we set double sensors at the \( z = 0 \) and \( z = \Delta z \) positions to record seismic data. Based on the double sensor data, a new migration scheme of the full acoustic wave equation on depth extrapolation is presented. The migration algorithm is easy to write, compute, program and implement. Here, several examples, including single shot and multiple shot examples, are introduced to test whether our method can obtain the true amplitude information of reflectors. After comparing the amplitude values obtained from the migration section with the theoretical reflection coefficients, our proposed method achieves satisfactory results. Additionally, in the actual example, the enhancement of the main reflection events is achieved by comparison with the migrated section after using the conventional Kirchhoff method.

Because double sensors are used in the scheme, there must be a perfect distance of double sensors that can recover the true amplitude while minimizing the design cost. A test model with the distance of the double sensor variable is built. After comparison of the calculated amplitudes of the reflector with the theoretical reflector coefficients, we found that using as small a distance as possible obtains better estimated values, matching well with the theoretical data. This finding may have practical significance in industrial design. It is a notable result to integrate double sensors into a single receiver device that will be easy to handle.

Beside the use of the double sensor technique in seismic migration, it can also be introduced into other seismic data processing fields, such as removing or utilizing ghost waves in future migration and providing new ideas about velocity analysis. Finally, it is worth mentioning that the usage of our double sensor acquisition technology in vector-acoustic reverse time migration and the unitization of the full-wave-equation depth migration to image the dual streamer or OBC data are worthwhile research directions in the future.

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Figure 10. (a) Comparison of picked amplitudes obtained from the reflector with theoretical reflection coefficients as the distance of dual sensor is 0.1 m; (b) the relative errors between calculated reflection coefficients and theoretical reflection coefficients in panel (a); (c) comparison of picked amplitudes obtained from the reflector with theoretical reflection coefficients as the distance of dual sensor is 0.3 m; (d) the relative errors between calculated reflection coefficients and theoretical reflection coefficients in panel (c); (e) comparison of picked amplitudes obtained from the reflector with theoretical reflection coefficients as the distance of dual sensor is 0.5 m; (f) the relative errors between calculated reflection coefficients and theoretical reflection coefficients in panel (e); (g) comparison of picked amplitudes obtained from the reflector with theoretical reflection coefficients as the distance of dual sensor is 0.7 m; (h) the relative errors between calculated reflection coefficients and theoretical reflection coefficients in panel (g); (i) comparison of picked amplitudes obtained from the reflector with theoretical reflection coefficients as the distance of dual sensor is 1.0 m; (j) the relative errors between calculated reflection coefficients and theoretical reflection coefficients in panel (i).
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Figure 11. (a) The migration section imaged by the conventional method; (b) the migration section imaged by our full-wave-equation migration method.

REFERENCES


