Apparent magnetization mapping in the presence of strong remanent magnetization: The space-domain inversion approach

Lianghui Guo¹, Lei Shi², Xiaohong Meng¹, Rui Gao³, Zhaoxi Chen¹, and Yuanman Zheng¹

ABSTRACT

Apparent magnetization mapping is a technique to estimate magnetization distribution in the subsurface magnetic layer from the observed magnetic data, of benefit in identifying lithologic units and delineating magnetic geologic boundaries. The conventional approaches for apparent magnetization mapping usually neglect effects of remanence, resulting in large geologic deviation and the occurrence of negative magnetization when the magnetic layer contains strong remanent magnetization. We have developed a space-domain inversion approach for apparent magnetization mapping based on the amplitude of magnetic anomaly (AMA), the analytic signal (AS), and the normalized source strength (NSS) to reduce effects of remanent magnetization. The AMA, AS, and NSS are three common quantities insensitive or weakly sensitive to the remanence transformed from the magnetic total field anomaly or components. The magnetic layer underground is first divided into a regular grid of vertical rectangular prisms, each having a cross-sectional area of one grid square and a uniform magnetization. Then, an iterative algorithm is adopted to invert each quantity of the AMA, AS, and NSS to obtain an optimum value of magnetization of each prism in the magnetic layer. The inversion approach permits the top and bottom surfaces of the magnetic layer to be constant or variable in depth, and requires no prior information of magnetization directions. Our tests on the synthetic and real data from the metallic ores area in the southern margin of North China have proved the feasibility and robustness of the presented inversion approach. All of the AMA, AS, and NSS inversions produced nonnegative magnetization distribution in the magnetic layer. Also, the AS and NSS inversions produced a better resolution of magnetization distribution than that of the AMA.

INTRODUCTION

Magnetization (or magnetization intensity) is one quantity of a physical property closely related to types of rocks and tectonic history. Apparent magnetization mapping (Grant, 1973; Misener et al., 1984; Silva and Hohmann, 1984; Pilkington and Crossley, 1987; Pilkington, 1989; Silva et al., 2010) is a technique to estimate magnetization distribution in the subsurface magnetic layer from the observed magnetic data. Rather than the true magnetization measured directly on a limited number of rock specimens, the magnetization estimated by this technique is an average of the rock magnetism within a certain depth below each observed station, and consequently it is called apparent magnetization. This technique is of benefit for identifying lithologic units and delineating magnetic geologic boundaries, and thus it has been applied for geologic mapping and tectonic studies and mineral exploration for decades.

The technique of apparent magnetization mapping usually models the magnetic layer as a collection of vertical, juxtaposed prisms in both horizontal directions, whose top and bottom surfaces are assumed to be horizontal or variable depth, and it then inverts or deconvolves the magnetic anomalies in the space or frequency domain to determine the magnetization of each prism. Grant (1973) presents a frequency-domain deconvolution approach for computing a magnetic susceptibility map, supposing that the top of the magnetic layer is located at a constant depth, whereas the bottom
is located at an infinite depth. Misener et al. (1984) propose a variable depth, space-domain approach for magnetization mapping, which considers a basement surface that usually varies in depth in the real world. Silva and Hohmann (1984) suggest reducing the magnetic anomalies to the pole and then computing the magnetization of an equivalent layer of poles at discrete points by a linear inversion procedure. Pilkington and Crossley (1987) use a Kalman filter to obtain estimates of the magnetization distribution when noise in the observed magnetic data is nonstationary. Pilkington (1989) presents a variable depth, frequency-domain approach for quick magnetization mapping that considers the variable depth of basement. This approach uses an iterative inversion scheme based on an approximation to the true partial derivative matrix for the linearized problem. Silva et al. (2010) adopt the entropic regularization algorithm to produce an estimated magnetization distribution with sharper boundaries, smaller volume, and higher apparent magnetization than those produced by conventional regularization algorithms, such as Tikhonov regularization.

However, one crucial problem of all the above approaches is that they assume that the magnetic sources contain no remanent magnetization and that the self-demagnetization effect can be ignored. Such assumptions are, however, not always valid because of the complicated nature of real geologic settings. For instance, the effect of remanent magnetization is often significant in volcanic terranes, metamorphic basements, basaltic oceanic crust, mafic or ultramafic intrusions, and metallic ores. In this case, the negligence of the remanence will result in a large geologic deviation or the occurrence of remanent magnetization. Such kinds of quantities include the analytic signal (AS), the amplitude of the magnetic total field anomaly (AMA), and the normalized source strength (NSS) is a quantity derived from the eigenvalues of the magnetic gradient tensors (MGTs), which represents a generalization of an expression derived by Wilson (1985) for the normalized magnetic moment of a dipole source. All three quantities of AS, AMA, and NSS were proved to be insensitive or weakly sensitive to magnetization direction, and their amplitude is only affected by the magnitude of the magnetization (Stavrev and Gerovska, 2000; Shearer, 2005; Li et al., 2010; Beiki et al., 2012; Clark, 2012, 2013); whereas the NSS compares very favorably with the other two quantities (Pilkington and Beiki, 2013).

In this paper, to reduce remanent magnetization effects on apparent magnetization mapping, we proposed space-domain inversion approaches based on each of the AMA, AS, and NSS. Similar to conventional mapping approaches, we consider the mapped model as one magnetic layer, whose top and bottom can be planar or undulating. The model is divided into a regular grid of vertical rectangular prisms, each having a cross-sectional area of one grid square and a uniform magnetization. Compared with unconstrained 3D inversion of the AMA (Li et al., 2010) and NSS (Pilkington and Beiki, 2013) or other inversions based on multiple layers model, the adoption of a one-layer model significantly reduces the number of prisms or cells composing the model, thus greatly reducing the requirement of physical memory and the amount of calculation. Unlike 3D inversions, the mapped magnetization distribution of one magnetic layer usually presents no dip information of magnetic sources. Because the AMA, AS, and NSS do not contain any dip information of magnetic sources (Li et al., 2010; Pilkington and Beiki, 2013), it is worth using them to map magnetization distribution for only one magnetic layer. The result of this kind of mapping theoretically is adequate to identify lithologic units and delineate magnetic geologic boundaries for geologic mapping and tectonic studies.

First we provide the forward calculation expressions of magnetic total field components, anomaly and gradients, and MGTs for a vertical rectangular prism. And then we present the forward calculation algorithms of AMA, AS, and NSS for a magnetic layer. Later, we put forward the iterative inversion algorithm based on the quantities of AMA, AS, and NSS for a magnetic layer. The top and bottom surfaces of the magnetic layer can be planar or undulating. No prior information of magnetization direction is required. Synthetical and real data from the metallic deposit area in the southern margin of North China are used to verify the feasibility of the new approach.

**METHODOLOGY**

**Magnetic forward calculation of a vertical rectangular prism**

For a given study area, we define a coordinate system with the \((x, y)\)-plane at sea level and the \(z\)-axis being positive downward. Suppose that in the subsurface, an arbitrary vertical rectangular
prism is centered at \((x_0, y_0, z_0)\); its widths along the \(x\)-, \(y\)-, and \(z\)-directions are, respectively, \(a\), \(b\), and \(c\); and its magnetization (intensity) is \(J\). The inclination and declination of the geomagnetic field are \(I_0\) and \(A_0\), respectively, whereas those of the total magnetization are \(I\) and \(A\), separately. Then, the components of the total field at an arbitrary station \((x, y, z)\) on the observational surface caused by the prism can be calculated by the following equation (Luo and Yao, 2007):

\[
B_x = J b_x, \quad B_y = J b_y, \quad B_z = J b_z, \quad (1)
\]

where \(\mu_0\) is the vacuum magnetic permeability; \((\xi, \eta, \zeta)\) is the coordinate of a 3D element volume throughout the volume of the prism, \(b_i (i = x, y, z)\) is the geometric function quantifying the contribution of a unit of magnetization in the prism to the observed station; and \(r = [(\xi-x)^2 + (\eta-y)^2 + (\zeta-z)^2]^{1/2}\). \(L = \cos I \cos A\), \(M = \cos I \sin A\), and \(N = \sin I\).

The theoretical magnetic total field anomaly at an arbitrary station \((x, y, z)\) on the observational surface caused by the prism can be calculated by the following equation (Luo and Yao, 2007):

\[
F = B_x L_0 + B_y M_0 + B_z N_0
= \mu_0 J \left[\begin{array}{c}
\frac{-k_1}{4\pi} \arctan \left(\frac{\xi - x}{\eta - y + \zeta - z + r}\right) \\
-k_2 \arctan \left(\frac{\zeta - x + \zeta - z + r}{\xi - x + \eta - y + r}\right) \\
-k_3 \arctan \left(\frac{\eta - y + \zeta - z + r}{\xi - x + \zeta - z + r}\right) \\
+k_4 \ln(\xi - x + r) + k_5 \ln(\eta - y + r) \\
+k_6 \ln(\zeta - z + r)
\end{array}\right],
\]

where \(L_0 = \cos I_0 \cos A_0, M_0 = \cos I_0 \sin A_0, N_0 = \sin I_0, k_1 = 2L_0, k_2 = 2M_0, k_3 = 2N_0, k_4 = N_0 + M_0, k_5 = N_0 + L_0, \) and \(k_6 = M_0 + L_0\). Equation 3 is valid when the magnetic total field anomaly is less than the geomagnetic field.

The theoretical magnetic total field gradients along the \(x\)-, \(y\)-, and \(z\)-directions at an arbitrary station \((x, y, z)\) on the observational surface caused by the prism can be calculated by the following equation (Luo and Yao, 2007):

\[
\frac{\partial F}{\partial x} = J f_x, \quad \frac{\partial F}{\partial y} = J f_y, \quad \frac{\partial F}{\partial z} = J f_z.
\]

By taking the \(x\)-, \(y\)-, and \(z\)-derivatives of the above magnetic total field components (equation 1), the theoretical MGTs at the station \((x, y, z)\) caused by the prism can be derived as follows:

\[
\text{MGT} = \begin{bmatrix}
B_{xx} & B_{xy} & B_{xz} \\
B_{yx} & B_{yy} & B_{yz} \\
B_{zx} & B_{zy} & B_{zz}
\end{bmatrix} = J \begin{bmatrix}
q_{xx} & q_{xy} & q_{xz} \\
q_{yx} & q_{yy} & q_{yz} \\
q_{zx} & q_{zy} & q_{zz}
\end{bmatrix},
\]

where \(B_{\alpha\beta}\) (\(\alpha = x, y, z; \beta = x, y, z\)) is one single MGT and \(q_{\alpha\beta}\) is the geometric function quantifying the contribution of a unit of magnetization in the prism to the observed station. The MGT is equivalent to the second-order gradient tensors of the pseudogravitational potential due to the source (in this case a prism) shown by Clark et al. (1998).

### Magnetic forward calculation of an arbitrary magnetic layer

In the subsurface, an arbitrary magnetic layer (see Figure 1) assumedly contains a heterogeneous magnetization with variable total magnetization intensities and directions. The top and the bottom surfaces of the magnetic layer can be plane or undulated. Suppose that the total magnetization intensities and directions, along with the depths of the top and the bottom surfaces, are known.

To do the forward calculation, the magnetic layer is first divided into a regular grid of vertical rectangular prisms, each having a cross-sectional area of one grid square and a uniform magnetization (see Figure 1). The magnetization of the \(j\)th prism is \(J_j\).


(j = 1, \ldots, m, where m is the total number of the prisms). Then, at an arbitrary \( i \)th station \((i = 1, \ldots, n, \text{where} \ n \text{is the total number of the observed stations})\) on the observational surface, the theoretical magnetic total field components caused by the magnetic layer can be expressed by a superposition of the theoretical magnetic total field components due to all the subsurface prisms:

\[
B_{x,i} \approx \sum_{j=1}^{m} J_j b_{x,ij}, \quad B_{y,i} \approx \sum_{j=1}^{m} J_j b_{y,ij}, \quad B_{z,i} \approx \sum_{j=1}^{m} J_j b_{z,ij}.
\]  

(8)

The theoretical magnetic total field gradients at the \( i \)th station caused by the magnetic layer can be expressed as follows:

\[
\left( \frac{\partial F}{\partial x} \right)_i \approx \sum_{j=1}^{m} J_j f_{x,ij}, \quad \left( \frac{\partial F}{\partial y} \right)_i \approx \sum_{j=1}^{m} J_j f_{y,ij}, \quad \left( \frac{\partial F}{\partial z} \right)_i \approx \sum_{j=1}^{m} J_j f_{z,ij}.
\]  

(9)

The theoretical MGT at the \( i \)th station caused by the magnetic layer can be expressed as follows:

\[
B_{nq,i} \approx \sum_{j=1}^{m} J_j b_{nq,ij}.
\]  

(10)

Then, the theoretical AMA at the \( i \)th station caused by the magnetic layer can be expressed as follows:

\[
\text{AMA}_i = \sqrt{B_{x,i}^2 + B_{y,i}^2 + B_{z,i}^2}.
\]  

(11)

The theoretical AS at the \( i \)th station caused by the magnetic layer can be expressed as follows:

\[
\text{AS}_i = \sqrt{\left( \frac{\partial F}{\partial x} \right)_i^2 + \left( \frac{\partial F}{\partial y} \right)_i^2 + \left( \frac{\partial F}{\partial z} \right)_i^2}.
\]  

(12)

Supposing the eigenvalues of matrix \( \text{MGT}_i \) in a monotone decreasing order are \( \lambda_1,i, \lambda_2,i, \text{and} \lambda_3,i \), respectively, then according to the definition of the NSS (Wilson, 1985; Beiki et al., 2012; Clark, 2012, 2013), the theoretical NSS at the \( i \)th station on the observational surface caused by the magnetic layer can finally be given by the following equation:

\[
\mu_i = \sqrt{-\lambda_2,i^2 - \lambda_1,i \lambda_3,i}.
\]  

(13)

The former studies (Wilson, 1985; Beiki et al., 2012) proved that the NSS value calculated from the above equation is always positive.

Theoretically, all of the AS, AMA, and NSS caused by the magnetic layer are independent or weakly dependent of magnetization direction, and their amplitude is only affected by the magnitude of the magnetization (Beiki et al., 2012; Clark, 2012, 2013). Hence, they are suitable to reduce remanence effects on magnetization mapping.

**Magnetization mapping of a magnetic layer based on the AMA, AS, and NSS**

Suppose that the top and bottom depths of the magnetic layer are known, whereas the magnetization intensities and directions are unknown. Then, the magnetization mapping based on the AMA, AS, or NSS tries to invert the magnetization of each prism in the magnetic layer from each of these three quantities. Because all three quantities are independent or weakly dependent of the magnetization direction, the vertical magnetized direction is assumedly considered as the total magnetization in the following procedure.

Take the NSS inversion as an example. Before performing the magnetization mapping, the NSS should be calculated from the observed MGT data using equation 13. If the observed MGT data are not available, they can be calculated from the observed magnetic total field components or anomalies using frequency-domain filters (Blakely, 1995). Then, this calculated NSS is considered as the observed one and is used to invert the magnetization of each prism in the magnetic layer.

To do the NSS inversion, a simple, iterative algorithm in the space domain proposed by Cordell and Henderson (1968) is used to obtain the optimum value of the magnetization intensity of each prism. Let \( J_{k,j} \) be the magnetization of the \( j \)th prism at the \( k \)th iteration. The algorithm seeks a set of prisms such that

\[
\lim_{k \to \infty} J_{k,j} = J_j, \quad \text{for all} \ j.
\]  

(14)

The following steps outline the procedure of the iterative algorithm:

**Step 1** Start the iteration by setting the initial magnetization of each prism, an arbitrary positive value. Here, we assumed an initial magnetization of each prism that is proportional to the observed NSS directly above that prism, and thus setted \( J_{0,j} = \mu_t \) (A/m). According to equation 10, the MGT \( \text{MGT}_{\text{cal},0,i} \) of the first approximation model at the \( i \)th station can be calculated. And then the NSS \( \mu_{\text{cal},0,i} \) of the first approximation model at the \( i \)th station can be calculated according to the eigenvalues of matrix \( \text{MGT}_{\text{cal},0,i} \) and equation 13.

**Step 2** Estimate the next modification of the magnetization of each prism, i.e., \( \{ J_{1,j} \} \), by the defining relationship:

\[
J_{1,j} = J_{0,j} \left( \frac{\mu_{\text{obs},i}}{\mu_{\text{cal},0,i}} \right),
\]  

(15)

or in general at the \( k \)th iteration:
Because the value of NSS is positive, the ratio between the observed NSS and the calculated one will be nonnegative, always making the modified magnetization nonnegative.

Step 3) Renew the MGT ($\text{MGT}_{\text{cal},k;i}$) of the subsurface model at the $i$th station according to equations 10 and 16, and then renew the NSS ($\mu_{\text{cal},k;i}$) at the $i$th station according to the eigenvalues of matrix $\text{MGT}_{\text{cal},k;i}$ and equation 13.

Step 4) Calculate the least-squares deviation between the observed NSS and the calculated NSS for the $k$th iteration and is given by the following equation:

$$
\phi = \sum_{i=1}^{n} (\mu_{\text{obs},i} - \mu_{\text{cal},k;i})^2.
$$

The goal of the iteration is to minimize the deviation $\phi$.

Step 5) If the deviation falls within a specified tolerance or the iteration number reaches the specified maximum, the iteration stops, and $J_{k,j}$ is considered as the final estimate of the magnetization of each prism. Otherwise, repeat steps (2–5).

In the entire procedure, the largest computation is the calculation of $b_{\alpha\beta,ij}$ for the $\text{MGT}_{\text{cal},k;i}$ in steps (1) and (3). However, it is recommended that this calculation only be performed one time, in step (1), and then it can be stored in the physical memory or written as a file, and then be called for the calculation of the $\text{MGT}_{\text{cal},k,i}$ in step
APPLICATION TO SYNTHETIC DATA

The synthetic magnetic layer is composed of six vertical rectangular prisms (Figure 2), which is similar to the model used by Pilkington and Beiki (2013) but having more complicated magnetization. The six prisms have various sizes, top and bottom depths, and locations, and due to the remanent magnetization, their total magnetization intensities and directions differ from each other (Table 1). Suppose that the inclination and declination of the geomagnetic field are, respectively, 90° and 0°. The observed geometry is a 66 × 66 regular grid with grid spacing of 1000 m at an altitude of 0 m. We forward calculated the theoretical magnetic total field anomalies and the theoretical AMA, AS, and NSS of the synthetic model, shown in Figure 2a–2d, respectively. Obviously, the theoretical magnetic total field anomalies are not consistent with the true prisms due to the effects of remanence, whereas all of the theoretical AMA, AS, and NSS are consistent with the true prisms, implying that they are insensitive or weakly sensitive to the remanence. Wherein, the AS and NSS belong to a quantity of first-order derivative, having a higher resolution than the AMA. As a whole, the NSS presents a most accordance to the true prisms. Hence, to do apparent magnetization mapping on the synthetic model, it is significant to adopt the approach based on the AMA, AS, and NSS to reduce the effects caused by the remanence.

We have considered the above theoretical AMA, AS, and NSS of the synthetic model as the observed ones, and we then respectively inverted each of them for the magnetization distribution of the magnetic layer using the presented approach. In this case, due to the unknown remanent magnetization, the inclination of 90° and declination of 0° were chosen as the total magnetization directions. In addition, because the uppermost depth of the six prisms in the synthetic model is 4 km and the deepest depth is 10 km, a flat top surface of the magnetic layer with depth of 4 km and a flat bottom with depth of 10 km were chosen for the inversions. Figure 3 describes the convergence curves of the three inversions after 10 iterations. The standard deviation of each inversion decreases quickly at the former five iterations and then slowly at the later iterations.

Figure 4 shows the magnetization maps produced by the inversions of the AMA, AS, and NSS after 10 iterations and their corresponding calculated AMA, AS, and NSS. Figure 5 displays their comparisons along two profiles of easting = 24 km and northing = 30 km. Apparently, all three inversions presented non-negative magnetization, and their distributions of high magnetization are close to the true prisms, implying an effective reduction of the effects of remanent magnetization. Among them, the AMA inversion produced the most smooth result with the relatively weakest magnitude of magnetization, whereas the NSS inversion produced the most compact result with the strongest magnitude of magnetization. The prisms of A and D are resolved clearly in all three results, whereas the other prisms, especially those of B and E, are not resolved clearly in the AMA and AS results but are resolved relatively clearly in the NSS result.

To test the sensitivity of the three inversions to the top and the bottom depths of the magnetic layer, we then did the inversion tests again by choosing a different top depth and bottom depth, respectively. Figure 6a–6c shows the magnetization maps produced by the inversions of the AMA, AS, and NSS after 10 iterations, when the top depth is 5 km and the other parameters are the same as in the first test. Figure 6d–6f shows those when the bottom depth is 11 km, and the other parameters are the same as the first test. The colorbar in Figure 6 is chosen as the same as that in Figure 4 for comparison. All of these magnetization maps are quite similar to the results of the first test, except for the magnitude of the mapped magnetization. Those produced by the test, when the top depth is 5 km (Figure 6a–6c), are a little stronger than those of the first test, due to deepening and thinning of the magnetic layer. Whereas those produced by the test when the bottom depth is 11 km are similar to those of the first test. Hence, the three inversions are sensitive to the top depth but not to the bottom depth. Despite the possible choice of imprecise top and bottom depths, the magnetization maps produced by the three inversions could still describe lithologic units and boundaries of magnetic sources.
Figure 4. The mapped magnetization distribution (a) by the AMA inversion and (b) the corresponding calculated AMA. The mapped magnetization distribution (c) by the AS inversion and (d) the corresponding calculated AS. The mapped magnetization distribution (e) by the NSS inversion and (f) the corresponding calculated NSS. The black dashed boxes in each map outline the locations of six vertical rectangular prisms.
To test the sensitivity of the three inversions to the noise, we contaminated the theoretical AMA, AS, and NSS (shown in Figure 2) of the synthetic model by Gaussian random noise of 4% of the datum magnitude, and then we respectively inverted each of them for the magnetization distribution of the magnetic layer by using the presented approach again. In this case, the parameters for the total magnetization directions and the depths of the top and bottom surfaces are the same as the first test.

Figure 7 shows the noisy AMA, AS, and NSS and their mapped magnetization distributions after 10 iterations. Figure 8 displays their comparisons along two profiles of easting = 24 km and northing = 30 km. The high-magnetization distributions produced by the three inversions are still close to the true prisms, implying an effective reduction of remanence effects under the interference of noise. Among them, the NSS inversion produced the most compact result with the strongest magnitude of magnetization. However, high-frequency noise in all the three inverted results will disturb the identification of magnetic sources. Hence, denoising is necessary in practical applications of magnetization mapping based on the AMA, AS, and NSS.

**APPLICATION TO REAL DATA**

The real magnetic total field anomalies data are from a metallic deposit area in the southern margin of North China. The near surface in this area is composed of the Mesozoic sediment cover and Archean metamorphic terrane at depth. The 23 boreholes in the studied area revealed that the sedimentary basement is relatively flat and the Archean metamorphic terrane at depth. The 23 boreholes in the studied area revealed that the sedimentary basement is relatively flat and the Archean metamorphic terrane at depth. The 23 boreholes in the studied area revealed that the sedimentary basement is relatively flat. The inclination and declination of the geomagnetic field are 52.14° and −5.23°, respectively. Due to remanent magnetization presented in the metallic ores and the metamorphic terrane, the magnetization intensities and directions of these magnetic sources differ greatly from each other. Hence, it is difficult to invert for correct magnetization distribution of the subsurface using the conventional approaches based on the magnetic total field anomalies or reduced-to-the-pole (RTP) anomalies. Here, we used the presented approach of apparent magnetization mapping based on the AMA, AS, and NSS for the test.

First, we did preferential filtering (Guo et al., 2013) on the real magnetic total field anomalies to obtain the residual magnetic anomalies (shown in Figure 9a), which mainly responds to the metallic ores in the near surface. The radial average power spectrum analysis (Blakely, 1995) showed that the central depth of the magnetic layer corresponding to the residual anomalies is approximately 1041 m. We then transformed the residual anomalies into the quantities of AMA, AS, and NSS (respectively shown in Figure 9b–9d). Obviously, the quantities of AMA, AS, and NSS are analogous regionally in shape, except that the AS and NSS have more detailed high-frequency information than does the AMA. One median size northwest-trending closure with strong magnitude is presented in the northwest of all the three quantities, consistent with the known buried metallic ores (the white bodies in Figure 9) revealed by the boreholes. Another large size northwest-trending closure with a median magnitude is presented in the southeast of all the three quantities.

We then considered the above transformed AMA, AS, and NSS as the observed ones, and we respectively inverted each of them for the magnetization distribution in the near surface using the presented approach. In this case, the inclination of 90° and declination of 0° were chosen as the total magnetization directions. According to the findings of the boreholes and the radial average power spectrum analysis of the magnetic anomalies, a flat top surface with a depth of 500 m and a flat bottom surface with a depth of 2000 m were chosen for the inversions.

Figure 10 shows the magnetization maps produced by the inversions of the AMA, AS, and NSS after 10 iterations and their corresponding calculated AMA, AS, and NSS. Figure 11 displays their...
Figure 6. The mapped magnetization distributions by the (a) AMA, (c) AS, and (e) NSS inversions, when the top depth of the magnetic layer is 5 km. The mapped magnetization distributions by the (b) AMA, (d) AS, and (f) NSS inversions when the bottom depth of the magnetic layer is 11 km. The black dashed boxes in each map outline the locations of six vertical rectangular prisms.
Figure 7. (a) The noisy AMA of the synthetic model and (b) its mapped magnetization distribution, (c) the noisy AS and (d) its mapped magnetization distribution, and (e) the noisy NSS and (f) its mapped magnetization distribution. The black dashed boxes in each map outline the locations of six vertical rectangular prisms.
Figure 8. Comparisons of the mapped magnetization distributions by the inversions of the noisy AMA (gray solid curve), AS (black dashed curve), and NSS (black solid curve) after 10 iterations along two profiles of (a) easting = 24 km and (b) northing = 30 km. Maps in panels (c) and (d), respectively, show the true outline of prisms and their magnetization intensities along profiles of easting = 24 km and northing = 30 km.

Figure 9. (a) The residual magnetic total field anomalies from one metallic deposit area in the southern margin of North China, and the (b) transformed AMA, (c) AS, and (d) NSS. The white bodies describe the known metallic ores in the near surface revealed by dozens of boreholes. The black line displays the location of profiles A-B for comparisons of different inversions as shown in Figure 10.
Figure 10. The mapped magnetization distribution (a) by the AMA inversion and (b) the corresponding calculated AMA. The mapped magnetization distribution (c) by the AS inversion and (d) the corresponding calculated AS. The mapped magnetization distribution (e) by the NSS inversion and (f) the corresponding calculated NSS. The white bodies describe the known metallic ores in the near surface revealed by dozens of boreholes.
comparisons along profiles A-B. Apparently, their distributions of high magnetization are analogous regionally in shape, except that the AS and NSS inversions presented more strong and high-resolution magnetization than did the AMA inversion. One median size northwest-trending closure of strong magnetization is presented in the northwest of all the three quantities, consistent with the known buried metallic ores there. Because the distance (1500 m) between the top and bottom surfaces of the mapped magnetic layer is much larger than the cumulative thickness (<100 m) of the metallic ores, it is reasonable that the magnitude of the mapped magnetization is generally a little weaker than the true magnetization magnitude of the metallic ores.

CONCLUSIONS

We have presented the space-domain inversion approach for apparent magnetization mapping based on the quantities of AMA, AS, and NSS to reduce effects of remanent magnetization. The forward calculation expressions of magnetic total field components, anomaly and gradients, and MGTs for a vertical rectangular prism were presented. The forward calculation algorithms of the AMA, AS, and NSS for an arbitrary magnetic layer and the procedures of the AMA, AS, and NSS inversions for apparent magnetization of the magnetic layer were proposed in detail. The top and bottom surfaces of the magnetic layer are permitted to be of the same depth or to be variable in depth. No prior information of magnetization directions is required for the inversions. Both tests on the synthetic and real data from the metallic ores area verified the feasibility and robustness of the presented approach. All the AMA, AS, and NSS inversions produced nonnegative magnetization distribution in the magnetic layer, and the latter two produced a higher resolution of magnetization map than the former one. As the calculations of AS and NSS involve transformation of first-order derivatives and thus are sensitive to high-frequency noise, denoising is necessary in practical applications of magnetization mapping based on the AS and NSS.

ACKNOWLEDGMENTS

We greatly thank the editor M. D. Sacchi, assistant editor J. C. Shragge, the associate editors C. Cevallos, M. Beiki, M. Pilkington, and one other anonymous reviewer for their helpful comments and valuable suggestions. This work was financially supported by the National Natural Science Foundation of China (41374093, 41404063), the SinoProbe Projects (SinoProbe-01-05 and SinoProbe-02-01), Beijing Higher Education Young Elite Teacher Project (YETP0650), the Fundamental Research Funds for the Central Universities (2652015214), the Major National Scientific Research and Equipment Development Project (ZDYZ2012-1-02-04), and the National 863 Project of China (2014AA06A613).

REFERENCES


Figure 11. Comparisons of the mapped magnetization distributions by the inversions of the AMA (gray solid curve), AS (black dashed curve), and NSS (black solid curve) after 10 iterations along profiles A-B.


