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3D correlation imaging of the vertical gradient of gravity data

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Abstract

We present a new 3D correlation imaging approach for vertical gradient of gravity data for deriving a 3D equivalent mass distribution in the subsurface. In this approach, we divide the subsurface space into a 3D regular grid, and then at each grid node calculate a cross correlation between the vertical gradient of the observed gravity data and the theoretical gravity vertical gradient due to a point mass source. The resultant correlation coefficients are used to describe the equivalent mass distribution in a probability sense. We simulate a geological syncline model intruded by a dike and later broken by two vertical faults. The vertical gradient of gravity anomaly of the model is calculated and used to test the approach. The results demonstrate that the equivalent mass distribution derived by the approach reflects the basic geological structures of the model. We also test the approach on the transformed vertical gradient of real Bouguer gravity data from a geothermal survey area in Northern China. The thermal reservoirs are located in the lower portion of the sedimentary basin. From the resultant equivalent mass distribution, we produce the depth distribution of the bottom interface of the basin and predict possible hidden faults present in the basin.

Keywords: gravity anomaly, vertical gradient, 3D correlation imaging, correlation coefficient, equivalent mass distribution

Introduction

Gravity exploration has been playing an important role in many fields of the earth sciences such as tectonic studies, resource exploration, and engineering and environmental problems. One important step in quantitative interpretation is the inversion of gravity data for estimating a mass or density-contrast model of the subsurface.

There have been two basic approaches to 3D gravity inversion. One approach fixes the geometric parameters of the model by subdividing the subsurface space into a set of rectangular cells, and then estimates the unknown density contrast of every cell automatically using some iterative optimization technique to minimize a specific target function (Braile et al. 1974, Bear et al. 1995, Li and Oldenburg 1998). However, this approach has the principal difficulty of the inherent nonunique solutions, because not only infinitely many models fit the known gravity data but also only a finite number of inaccurate gravity measurements are available. Meanwhile, in 3D, the approach requires a large amount of computer memory and computation time (Li and Oldenburg 2003, Yao et al. 2007). Numerous techniques have been developed for trying to deal with the nonuniqueness and computation problems. These include inversion with constraints of \textit{a priori} geological or density-contrast information or depth weighting (Bear et al. 1995, Li and Oldenburg 1998, Boulanger and Chouteau 2001), regularized focusing inversion (Portniaguine and Zhdanov 1999), compression techniques based on cubic interpolation (Portniaguine and Zhdanov 2002), wavelet transforms and the logarithmic barrier method (Li and Oldenburg 2003), inversion in stochastic subspaces (Yao et al. 2007), data-space
inversion with sparseness constraints (Pilkington 2009), and adaptive-learning procedures (Dias et al. 2009). The alternative approach assumes constant density-contrasts and estimates the unknown geometric parameters of the model either manually by trial and error (Talwani and Ewing 1960) or automatically by using some iterative optimization technique (Cordell and Henderson 1968, Oldenburg 1974, Gomzoritz and Agarwal 2005, Chakravarti and Sundararajan 2007). But this approach also has the principal problem of inherent nonuniqueness for the aforementioned reasons.

The 3D gravity probability tomography approach is a new imaging method proposed by Mauriello and Patella (2001a, 2001b). It calculates the Δ-mass occurrence probability function (Mauriello and Patella 2001a) at each 3D regular grid node in the subsurface, which implies the probability of a point mass source distributed at the node responsible for the observed data. Since equivalent point masses are known to be the deepest sources compatible with any gravity anomaly (Nettleton 1976), the approach can be used to locate maximum-depth anomalous sources (Mauriello and Patella 2001a). The approach has the advantage of requiring no a priori information for the tomography and demanding low amount of computer memory by using a variable size moving-window algorithm. However, the problem is that the approach is unable to distinguish complex anomalous sources with a complicated distribution except for simple and isolated sources. Juliano et al. (2002) later introduced integrated probability tomography for any pair of gravity, magnetic and self-potential data sets for an integrated interpretation. Alaia et al. (2008, 2009) presented the generalized probability tomography of gravity data by assuming that the gravity anomaly can be caused by a discrete number of monopoles, dipoles, quadrupoles and octopoles. The result of Alaia’s approach could be used to analyze the shape and position of the most probable minimum structure of the anomalous sources.

Based on the 3D gravity probability tomography approach, we present a 3D correlation imaging approach for vertical gradient of gravity data, which involves directly calculating the cross correlation between the vertical gradient of the observed gravity data and the theoretical gravity vertical gradient due to a point mass source at each node of a subsurface 2D regular grid. The resultant correlation coefficients are used to describe the equivalent mass distribution in a probability sense. Since the gravity gradient anomaly reflects the edges and shapes of anomalous sources rather than just mass distribution (Bell et al. 1998), our approach theoretically derives a higher-resolution equivalent mass distribution than 3D correlation imaging of gravity anomalies. Our approach is suited to be used in the early stages of the interpretation process, especially when a priori information is unavailable. We illustrate the approach with both synthetic and real examples.

**Method**

At a survey area, we choose a coordinate system with the (x, y)-plane at sea level and the z-axis positive downwards. Suppose that at an arbitrary point mass source \( q(x_q, y_q, z_q) \) is present in the subsurface, its volume is \( v_q \), and its density contrast is \( \Delta \sigma_q \), then the theoretical gravity anomaly at an arbitrary gravity station \((x, y, z)\) on the observational surface caused by the point mass \(q\) can be calculated by (Kearey et al. 2002)

\[
\Delta g_q(x, y, z) = G \Delta \sigma_q v_q B_q(x, y, z),
\]

where \(G\) is the universal gravitation constant, and \(B_q(x, y, z)\) is the geometrical function of the point mass \(q\) for gravity anomaly at the station \((x, y, z)\):

\[
B_q(x, y, z) = \frac{(z_q - z)}{[(x_q - x)^2 + (y_q - y)^2 + (z_q - z)^2]^{3/2}}.
\]

Calculating the vertical derivative of formula (1), we obtain the theoretical vertical gradient of gravity anomaly at the station \((x, y, z)\) due to the point mass \(q\) as

\[
\Delta g_q(x, y, z) = G \Delta \sigma_q v_q B_z,q(x, y, z),
\]

where \(B_z,q(x, y, z)\) is the geometrical function of the point mass \(q\) for the gravity vertical gradient at the station \((x, y, z)\):

\[
B_z,q(x, y, z) = 2 \frac{(z_q - z) - (x_q - x) - (y_q - y)}{[(x_q - x)^2 + (y_q - y)^2 + (z_q - z)^2]^{5/2}}.
\]

We define the correlation coefficient function between the vertical gradient of observed gravity anomaly and the theoretical gravity vertical gradient caused by the point mass \(q\) based on Pearson’s linear correlation formula as

\[
C_{z,q} = \frac{\sum_{i=1}^{N} \Delta g_z(x_i, y_i, z_i) \Delta g_z,q(x_i, y_i, z_i)}{\sqrt{\sum_{i=1}^{N} \Delta g_z^2(x_i, y_i, z_i) \sum_{i=1}^{N} \Delta g_z,q^2(x_i, y_i, z_i)}},
\]

where \(\Delta g_z(x_i, y_i, z_i)\) is the vertical gradient of observed gravity anomaly at the station \((x_i, y_i, z_i)\), and \(N\) is the total number of observed stations.

Supposing \(\Delta \sigma_q > 0\), then substituting formula (1) and (4) into formula (5) yields

\[
C_{z,q} = \frac{\sum_{i=1}^{N} \Delta g_z(x_i, y_i, z_i) B_z,q(x_i, y_i, z_i)}{\sqrt{\sum_{i=1}^{N} \Delta g_z^2(x_i, y_i, z_i) \sum_{i=1}^{N} B_z,q^2(x_i, y_i, z_i)}},
\]

According to the Cauchy inequality, we know that the correlation coefficient \(C_{z,q}\) in formula (6) satisfies the condition \(-1 \leq C_{z,q} \leq +1\).

The values of \(C_{z,q}\) reflect the cross correlation degree between the vertical gradient of observed gravity anomaly and the theoretical gravity vertical gradient due to the point mass \(q\). It implies the probability that an anomalous mass at the point \(q\) is responsible for the observed data. A positive value of \(C_{z,q}\) indicates that the anomalous mass is an excess, while a negative value of \(C_{z,q}\) means the anomalous mass is a deficiency. The closer the absolute value of \(C_{z,q}\) is to 1, the higher the probability of an excess or deficient mass at the point \(q\).

We call this approach 3D correlation imaging since it derives the subsurface equivalent mass distribution in a probability sense by calculating the cross correlation between
the vertical gradient of observed gravity anomaly and the theoretical gravity vertical gradient due to a point mass source.

To apply the 3D correlation imaging procedure for the vertical gradient of observed gravity anomaly, we first divide the subsurface space into a 3D regular grid. Then by using formula (6), we calculate the correlation coefficient of each node of the 3D regular grid from the top to the bottom in turn. This yields a 3D correlation coefficients dataset. By visualizing the dataset, we can describe the equivalent mass distribution according to the values of correlation coefficients.

The 3D correlation imaging approach can be easily expanded to gravity anomalies and their horizontal gradients. For example, the correlation coefficient function for gravity anomalies can be defined as

\[
C_q = \frac{\sum_{i=1}^{N} \Delta g(x_i, y_i, z_i) B_q(x_i, y_i, z_i)}{\sqrt{\sum_{i=1}^{N} \Delta g^2(x_i, y_i, z_i) \sum_{i=1}^{N} B^2_q(x_i, y_i, z_i)}},
\]  

(7)

where \(\Delta g(x_i, y_i, z_i)\) is the observed gravity anomaly at the station \((x_i, y_i, z_i)\), and \(B_q(x_i, y_i, z_i)\) is the geometrical function of the subsurface point mass \(q\) for gravity anomaly at the station \((x_i, y_i, z_i)\) as given in formula (2).

Comparing to the 3D gravity probability tomography approach (Mauriello and Patella 2001a, 2001b), our geometrical function in formula (2) is the same as the so-called space-domain scanning function of Mauriello and Patella (2001a, 2001b). Also our correlation coefficient function in formula (7) is similar to the so-called \(\Delta\)-mass occurrence probability function of Mauriello and Patella (2001a, 2001b), except that no topography surface regularization function (Mauriello and Patella 2001a) is involved in our formula. However, the integration surface in our correlation coefficient function is fixed as the whole survey area, while that in the \(\Delta\)-mass occurrence probability function is varied by progressively increasing the \(X\) and \(Y\) boundaries up to the size of the whole survey area (Mauriello and Patella 2001b).

Data experiments

In this section, we will test the 3D correlation imaging approach separately on synthetic gravity data and real gravity data.

Test on synthetic gravity data

The test model simulates the geological structure of a syncline with the intrusion of a dike along its axis. The inclination of the left flank of the syncline is 45°, while that of the right one is about 26.6°. The inclination of the dike is about 63.4°. The syncline is broken into three parts by two normal faults vertical to the strike of the syncline. Both the density contrast of the syncline flank and that of the dyke are 500 kg m\(^{-3}\). The density contrast of the other portion in the model is 0 kg m\(^{-3}\). Figure 1 shows the 3D visualization of the model with an extent of 3000 m along the \(X\)-axis, 3000 m along the \(Y\)-axis, and 600 m along the depth-axis. We do the forward modelling of the model for gravity anomaly and its vertical gradient on a flat surface with an elevation of 10 m. The observed geometry is a \(101 \times 101\) regular grid with a grid spacing of 30 m along both \(X\)-axis and \(Y\)-axis. Figure 2(a) displays the map of the gravity anomaly of the model contaminated by a Gaussian noise of 0.05 mGal and 2% of the datum magnitude. Figure 2(b) shows the map of the gravity vertical gradient of the model contaminated by a Gaussian noise of 1 Eötvös and 2% of the datum magnitude. Obviously the vertical gradient (figure 2(b)) has a higher resolution than the gravity anomaly (figure 2(a)), and reflects more details of the geological structures.

Since the density contrasts of the flank and dyke are positive, their masses are in excess and consequently their correlation coefficients are anticipated to be positive. Therefore, in order to make most of the resultant correlation coefficients of the approach positive, we produced new synthetic data by adding constants of −2.7 mGal and 45 Eötvös respectively to the synthetic gravity anomaly data (figure 2(a)) and the vertical gradient data (figure 2(b)), which makes the minimum values of both sets of data zero. Then we tested the 3D correlation imaging approach separately on the new synthetic gravity anomaly data and vertical gradient data with a depth step of 30 m and a depth extent of −600–0 m.

Figures 3(a)–(d) show the maps of the correlation coefficients calculated by the 3D correlation imaging approach for the synthetic gravity anomaly respectively along profiles A–A′, B–B′, C–C′ and D–D′ (shown with black wide solid
lines in figure 2(a)). The relatively low positive values of correlation coefficients in the centre portions of figures 3(a)–(c) and in the upper portion of figure 3(d) indicate a low probability of excess mass, while below them the high positive values of correlation coefficients means a high probability of excess mass. Comparing to the true syncline and the dyke depicted by the black wide solid lines in each map, the equivalent mass distribution is too smooth to reveal the true geological structures.

Figures 3(e)–(h) display the results of the vertical gradient data along profiles A–A’, B–B’, C–C’ and D–D’ respectively. They have a remarkably higher resolution and present more details of geological structures than figures 3(a)–(d). Supposing that the contour value of 0 of the correlation coefficients is to be the interface of the syncline, it is close to the interface of the true syncline (black wide solid lines in each map). This predicted syncline distributes shallow in profile A–A’ and deep in profile C–C’. The left flank of the
The small-scale positive values of correlation coefficients above the syncline in figures 3(h) reveal the faults in the model. The slopes between the steps near to (1000 m, −450 m) and (2000 m, −300 m) in figure 3(h) indicate the probability of excess mass distribution, which are actually due to the dike.

**Test on real gravity data**

The real gravity data are from a geothermal survey area in the west of the North China plain. Geologically, an east–west trending syncline characterizes this area, of which the flank is covered with strata of Triassic, Jurassic and Cretaceous age, and the core is a sedimentary basin with formations of Paleogene, Neogene and Quaternary age. The north of the syncline is bordered with the Cambrian and Ordovician strata by a normal fault that strikes approximately EW and dips southwards. The previous geothermal exploration in the nearby area found that the thermal reservoirs are primarily stored in the thick sandstone and mudstone of Paleogene age, and is covered by strata of Triassic, Jurassic and Cretaceous age. The syncline is steep while the right one is relatively gentle. From figure 3(h), we can see that the axis of the syncline looks like three steps, and is deep on the left and shallow on the right. The low positive and negative values of correlation coefficients below the interface in figures 5(f)–(j) indicate the low probability distribution of excess and deficient mass, which are actually due to the Cenozoic strata in the basin, which are of lower densities than the Mesozoic strata in the flank. Some obvious gradient zones of the gravity anomaly approximately match the locations of the known normal faults (white dotted lines in figure 4(a)). A shallow geothermal resource has been exploited in the fault zone around Easting 15 km and Northing 16 km, which is caused by deep water circulating through the deep fault. Figure 4(b) shows the vertical gradient of the Bouguer gravity anomaly transformed by the conventional Fourier transformation method.

Supposing that the rock of the lowest density in the subsurface is of zero density contrast, the Cenozoic strata are of low positive density contrasts, and the Mesozoic strata are of relatively high positive density contrasts. Then their masses are in excess and consequently their correlation coefficients are anticipated to be positive. Therefore, in order to make most of the resultant correlation coefficients of the approach positive, we produced new data by adding constants of 9.6 mGal and 29.4 Eötvös respectively to the Bouguer anomaly data (figure 4(a)) and the vertical gradient data (figure 4(b)), which makes the minimum values of both data sets zero. Then we tested the 3D correlation imaging approach on the new Bouguer gravity anomaly data and vertical gradient data respectively. The 3D regular grid for imaging is 116 × 84 × 26 nodes with a grid spacing 0.2 km in each direction. The depth range of the imaging is −4.9–0.1 km.

Figures 5(a)–(e) display the calculated correlation coefficients from the 3D correlation imaging of the Bouguer gravity anomaly respectively along profiles A–A′, B–B′, C–C′, D–D′ and E–E′ (shown with black wide solid lines in figure 4(a)). Figures 5(f)–(j) display those of the vertical gradient. The resultant equivalent mass distribution of the Bouguer gravity anomaly is monotonic and smooth, while that of the vertical gradient indicates significant local details. According to the nearby geothermal exploration to the east of the survey area, the maximum depth of the basin is about −6 km, and it becomes shallower from the axis to the edges. Hence, we suppose that the value of 0.12 of the correlation coefficients represents the bottom interface of the basin. The low positive and negative values of correlation coefficients above the interface in figures 5(f)–(j) indicate the low probability distribution of excess and deficient mass, which are actually due to the low density-contrast strata of Cenozoic age. On the other hand, the relatively high positive values of correlation coefficients below the interface indicate the high probability distribution of excess mass, which is due
Figure 5. The equivalent mass distribution from the 3D correlation imaging of the Bouguer anomaly along profiles A–A’ (a), B–B’ (b), C–C’ (c), D–D’ (d), and E–E’ (e), and those of the vertical gradient along profiles A–A’ (f), B–B’ (g), C–C’ (h), D–D’ (i), and E–E’ (j).

to the high density-contrast strata of Mesozoic age. Figure 6 shows the depth distribution of the interface. It presents an east–west trending, and is about 1.8 km deep in the south of the survey area, 3–5 km deep in the middle, and 2–3 km deep in the north. The axis of the interface is 3.5 km deep on average to the west, and 5 km or deeper from Easting 14 km to the east.

The slopes at Northing 6.5 and 11.5 km in profile A–A’ (figure 5(f)), Northing 3 and 14 km in profile B–B’ (figure 5(g)), Northing 13.5 km in profile C–C’ (figure 5(h)), and Northing 16 km in profile D–D’ (figure 5(i)), correlate with several known normal faults (white dotted lines in figure 4(a)). The steep slope at Easting 14 km in profile E–E’ (figure 5(j)) is predicted to be a hidden deep fault. From figure 6, we can see that this hidden fault is expanded to the depth of the exploited geothermal resource around Easting 15 km and Northing 16 km. This hidden fault is presumed to be an important channel for flow and deep circulation of underground water. The excess mass distribution at Northing 9 km in profile B–B’ (figure 5(g)) and Easting 4–10 km in profile E–E’ (figure 5(j)) indicate possible local uplifts, hidden faults, or local anomalous sources like intrusions.

The above brief predictions of geological structures will provide a reference for understanding and evaluating the geothermal resources in this area. Further geological and geophysical studies are needed to verify our predictions.

Conclusions

We have presented the 3D correlation imaging approach for vertical gradient of gravity anomaly. It calculates a cross
correlation between the vertical gradient of the observed gravity data and the theoretical gravity vertical gradient due to a point mass source at each node of the subsurface 3D regular grid. The resultant correlation coefficients are used to describe the subsurface equivalent mass distribution in a probability sense. The application to synthetic gravity data and real gravity data demonstrates that the resultant correlation coefficients of the approach can delineate geological structures or anomalous sources in the subsurface, and illustrates that the 3D correlation imaging for gravity vertical gradient produces a higher resolution of equivalent mass distribution than that for gravity anomalies alone. Therefore, high quality gravity data are required to obtain good gravity vertical gradients for correlation imaging.

The approach is simple, easy to perform, computationally stable with low requirements for computer memory, and insensitive to the noise in the observed data. It can do imaging stably for large-scale observed data. The approach is suited to be used in the early stages of the interpretation process for an early evaluation of the subsurface mass distribution, or locating maximum-depth anomalous sources, especially when no or little a priori information is available.

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