Prestack Kirchhoff time migration of 3D coal seismic data from mining zones

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\textbf{ABSTRACT}

Conventional seismic data processing methods based on post-stack time migration have been playing an important role in coal exploration for decades. However, post-stack time migration processing often results in low-quality images in complex geological environments. In order to obtain high-quality images, we present a strategy that applies the Kirchhoff prestack time migration (PSTM) method to coal seismic data. In this paper, we describe the implementation of Kirchhoff PSTM to a 3D coal seam. Meanwhile we derive the workflow of 3D Kirchhoff PSTM processing based on coal seismic data. The processing sequence of 3D Kirchhoff PSTM includes two major steps: 1) the estimation of the 3D root-mean-square (RMS) velocity field; 2) Kirchhoff prestack time migration processing. During the construction of a 3D velocity model, dip moveout velocity is served as an initial migration velocity field. We combine 3D Kirchhoff PSTM with the continuous adjustment of a 3D RMS velocity field by the criteria of flattened common reflection point gathers. In comparison with post-stack time migration, the application of 3D Kirchhoff PSTM to coal seismic data produces better images of the coal seam reflections.

\textbf{Key words:} Coal Seam, Post-stack, Prestack.

\section*{INTRODUCTION}

Although prestack time migration (PSTM) has already been an alternative to post-stack time migration in petroleum exploration for decades, because of relatively simple subsurface structures, post-stack time migration methods have been dominating coal seismic data processing. Conventional processing based on post-stack time migration is related to two crucial assumptions (Yilmaz and Claerbout 1980). The first assumption is that subsurface reflectors are horizontally layered media. The second assumption is that the common midpoint (CMP) stack is equivalent to a zero-offset section. The processing sequence includes normal moveout (NMO) correction, stack and migration. However, NMO corrections and stacking of CMP gathers of steeply dipping reflections usually cause midpoint smearing. Prestack partial migration (Yilmaz and Claerbout 1980) is therefore applied to common offset sections individually so that it can remove the effect of offset on dipping events and produce a more coherent stack. Prestack partial migration processes, when used in velocity analysis, can improve the estimation of seismic velocities needed for migration as the stacking velocities tend to be RMS type velocities (Bancroft, Geiger and Margrave 1998). This process is now referred to as dip moveout (DMO) (Deregowski and Rocca 1981; Hale 1984; Deregowski 1986). Due to limited computer resources, the application of the DMO process was not widely applied in the petroleum industry until the end of the 1980s. Although used for post-stack time migration, the DMO process prompted the development of prestack migration. Prestack migration algorithms are directly applied to
common-shot gathers or common-offset gathers rather than to zero-offset stacked sections. For complex geological areas, prestack migration can result in relatively reliable subsurface images.

Many migration methods have been developed since the 1970s. These methods can be classified into two major categories: 1) wave-propagation-migration (Claerbout 1976, 1985; Stolt 1978; Chun and Jacewitz 1981; Yilmaz 1987) and 2) Kirchhoff summation (integral) migration (Schneider 1978). Each class of migration algorithms has its own advantages and disadvantages. Although 3D wave-propagation-migration algorithms are also widely used in the oil and gas industry, these methods are still computationally expensive. With the successful application of the Kirchhoff integral method to 2D and 3D data from the Gulf of Mexico (Ratcliff, Gray and Whitmore 1992), 3D Kirchhoff prestack depth migration (PSDM) has been promoted in petroleum exploration since the 1990s. However, high-quality images of Kirchhoff PSDM rely on the construction of an accurate interval velocity model. Furthermore, the construction of an accurate velocity model is rather difficult and time-consuming process (Pon and Lines 2005). In structurally complex areas, 3D Kirchhoff PSDM sometimes fails to obtain accurate images. Since RMS velocity model building is especially simple compared with depth-migration methods (Alkhalifah 2006), it is easy to generate accurate images with 3D Kirchhoff PSTM using a RMS velocity model in comparison with 3D Kirchhoff PSDM using an interval velocity model. At present, the Kirchhoff PSTM process is still used in the oil and gas industry owing to its robustness, despite obvious limitations in handling lateral inhomogeneity.

So far, many reports on prestack migration algorithms in petroleum seismic data processing have been published. However, literature concerning the application of prestack migration to coal seismic data is relatively limited due to the simple coal geological structure. Recently there have been some problems in the imaging of 3D coal seismic data from mining zones. Since conventional data processing methods based on post-stack migration could not produce correct images from 3D coal seismic data, we propose the application of the Kirchhoff PSTM method to 3D coal seismic data. In this paper, we describe the iterative procedure of velocity estimation and migration in detail. We also illustrate the application of Kirchhoff PSTM to 3D coal seismic data. Comparison with post-stack time migration shows that Kirchhoff PSTM is an effective method for 3D coal seismic data. We hope the example given here can serve as a reference for the future use of this method in coal seismic data processing.

**KIRCHHOFF INTEGRAL THEORY**

By assuming homogeneous, isotropic and full-elastic media, the P-wave equation can be written as the following expression (Stolt 1978; Wen, McMechan and Booth 1988):

\[
\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + \frac{\partial^2 P}{\partial z^2} - \frac{1}{V^2} \frac{\partial^2 P}{\partial t^2} = 0.
\]  

(1)

Equation (1) is the acoustic wave equation where \(x, y, z\) and \(t\) denote spatial coordinates, \(t\) denotes time, \(V\) denotes P-wave velocity and \(P\) is the function of the wavefield, which is the function of \(x, y, z, t\).

The integral solution to equation (1) can be written as:

\[
P(x, y, z, t) = \frac{1}{4\pi} \int dS_0 \int dA_0 \times \left[ G \frac{\partial}{\partial n} P(x_0, y_0, z_0, t_0) - P(x_0, y_0, z_0, t_0) \frac{\partial}{\partial n} G \right],
\]  

(2)

where \(S_0\) is the surface of integration, \(P(x_0, y_0, z_0, t_0)\) is observed seismic data, \(P(x, y, z, t)\) is the solution to the wavefield of the observed \(r(x, y, z)\) on \(S_0\), \(n\) is the outward normal vector to the surface \(S_0\) and \(G\) is the Green’s function for a point source at \(r(x, y, z)\).

By choosing the enclosing surface \(S_0\), which consists of \(A\) and \(A_0\), where \(A_0\) is the observation plane on the surface \(S_0\) and \(A\) is a hemisphere extending to infinity in the subsurface, contributions from the distant hemisphere are ignored and equation (2) is expressed by

\[
P(x, y, z, t) = \frac{1}{4\pi} \int dS_0 \int dA_0 \times \left[ G \frac{\partial}{\partial n} P(x_0, y_0, z_0, t_0) - P(x_0, y_0, z_0, t_0) \frac{\partial}{\partial n} G \right].
\]  

(3)

The Green’s function with the desired properties on the free surface consists of a point source and its negative image. It is given as

\[
G(x, y, z, t|x_0, y_0, z_0, t_0) = \frac{\delta(t - t_0 - \frac{R}{V})}{R} - \frac{\delta(t - t_0 - \frac{R'}{V})}{R'},
\]  

(4)

where

\[
R = \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2},
\]

\[
R' = \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z + z_0)^2}.
\]

Due to \(z_0 = 0\) on the observation plane \(A_0\), we get \(R = R'\). Substituting \(R = R'\) into equation (4), we have \(G = 0\). Thus, equation (3) can be given as

\[
P(x, y, z, t) = -\frac{1}{4\pi} \int dS_0 \int dA_0 \left[ P(x_0, y_0, z_0, t_0) \frac{\partial}{\partial n} G \right].
\]  

(5)
Since the outward normal vector to the surface $S_0$ is in an opposite direction to coordinate $z_0$, we get

$$\frac{\partial G}{\partial n} = -\frac{\partial G}{\partial z_0}. \quad (6)$$

Substituting equations (4) and (6) into equation (5) and with simplification yields the following integral representation for the wavefield $P(x, y, z, t)$ at any point in the image space in terms of observations of the wavefield $P(x_0, y_0, z_0, t_0)$ on the surface,

$$P(x, y, z, t) = \frac{1}{2\pi} \int dt_0 \int dA_0 \times \left[ P(x_0, y_0, z_0, t_0) \frac{\partial}{\partial z_0} \frac{\delta(t - t_0 - z_0)}{R} \right]. \quad (7)$$

Equation (7) is commonly called the Kirchhoff integral (Schneider 1978).

We derive the following equation (Appendix A) through downward extrapolation of the transformation of equation (7),

$$P(x, y, z, t) = -\frac{1}{2\pi} \frac{\partial}{\partial z} \int dA_0 P(x_0, y_0, 0, t + \frac{z}{R}). \quad (8)$$

Equation (8) is the far-field approximation of the integral for the homogeneous case.

According to the imaging principle that extrapolates receivers for all $z > 0$ at $t = 0$, equation (8) is given as

$$P(x, y, z, 0) = -\frac{1}{2\pi} \frac{\partial}{\partial z} \int d\xi d\eta P(x_0, y_0, 0, \frac{\xi}{R}). \quad (9)$$

The solution to the wavefield in equation (9) can yield 3D migration. Integral equation (9) can be written as follows:

$$P(m\Delta x, n\Delta y, l\Delta z, 0) = -\frac{1}{2\pi} \frac{\Delta}{\Delta z} \sum_{\xi=-N}^{M} \sum_{\eta=-N}^{M} \frac{P(\xi\Delta x, \eta\Delta y, 0, \frac{\xi}{R})}{\sqrt{(m-\xi)^2\Delta x^2 + (n-\eta)^2\Delta y^2 + l^2\Delta z^2}} \Delta x \Delta y, \quad (10)$$

$m = 0, \pm 1, \pm 2, \ldots, \pm M,$

$n = 0, \pm 1, \pm 2, \ldots, \pm N,$

$l = 0, \pm 1, \pm 2, \ldots, \pm L,$

where $\Delta x, \Delta y, \Delta z$ are the space sampling along the $x, y, z$ axis, respectively, $m, n, l$ denote the sample ordinal number of subsurface points along the $x, y, z$ axis, respectively, $\xi, \eta$ are the sample ordinal number of surface points along the $x, y$ axis, respectively and $\frac{\Delta}{\Delta z}$ is the one-order difference along coordinate $z$.

By summing seismic data with equation (10) we obtain 3D migration based on the Kirchhoff integral formula.

**SURVEY TARGET AND DATA ACQUISITION**

Survey target

The study region is a coal mine with an area of nearly 10 km$^2$, located in the south of China. Three coal seams have already been detected. The average thickness of each coal seam ranges from 3–5 m. So far, two sets of 3D seismic data have been acquired in this area in 2001 and 2005, respectively.

Figure 1 is a post-stack time migrated section through the mining zones, extracted from the first 3D seismic data volume in the cross-line direction. Figure 1 shows the configuration of the subsurface coal seams before mining. The structural features of all coal seam reflections appear nearly horizontally continuous in Fig. 1 and have already been verified by drilling. The reflection events of the three horizontal coal seams are marked in Fig. 1. Coal seam 1, also marked by $S_1$ with a red line is located at about 490 ms, coal seam 2 marked by $S_2$ with a green line is around 530 ms and coal seam 3 marked by $S_3$ with a blue line is at about 580 ms. Since part of coal seam 1 was taken away earlier in 2003, the top of coal seam 1 collapsed. In order to detect the damage of coal seams, a second 3D seismic data acquisition covering the mining zones was conducted in 2005.
Table 1 Second data acquisition parameters

<table>
<thead>
<tr>
<th>Items</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source</td>
<td>3.0 kg dynamite</td>
</tr>
<tr>
<td>Source depths</td>
<td>12 m</td>
</tr>
<tr>
<td>Receiver</td>
<td>100-Hz geophone</td>
</tr>
<tr>
<td>Number of channels</td>
<td>$48 \times 8$</td>
</tr>
<tr>
<td>Sampling rate</td>
<td>1 ms</td>
</tr>
<tr>
<td>Record length</td>
<td>2000 ms</td>
</tr>
<tr>
<td>Receiver spacing</td>
<td>$20 \text{ m} \times 40 \text{ m}$</td>
</tr>
<tr>
<td>Source spacing</td>
<td>$20 \text{ m} \times 80 \text{ m}$</td>
</tr>
<tr>
<td>CMP bin size</td>
<td>$10 \text{ m} \times 10 \text{ m}$</td>
</tr>
<tr>
<td>CMP Fold</td>
<td>32</td>
</tr>
<tr>
<td>Offset ranges</td>
<td>10–950 m</td>
</tr>
</tbody>
</table>

Data acquisition

In the first data acquisition, 2.0 kg of dynamite was used as the seismic source with a source depth of 10 m and the seismic data were collected to produce a 24-fold CMP seismic profile. All records consist of 1500 samples with a 1 ms sample interval. The area of the first 3D survey is 15 km². Due to the mining zones being the study target in the second 3D survey, the area of the 3D survey is only 0.8 km² and the total number of shotpoints is 900. In order to acquire high-quality seismic data, 3.0 kg of dynamite was served as the seismic source with a source depth of 12 m and 32-fold CMP in the second data acquisition. All records possess 4000 samples with a 0.5 ms time sampling interval and a record length of 2000 ms. Table 1 gives the second data acquisition parameters. Unfortunately, since the company did not think about 4D exploration in the acquisition design, the shot and receiver locations are slightly different between the two 3D surveys with an approximate 8 degree angle difference.

The field data shown in Fig. 2 are part of a shot gather from the second 3D survey in 2005. Clearly visible reflection events appear as the dominant signals above 500 ms. Ground roll exhibits below 300 ms and it obscures reflection events that might be presented in near offsets of every split-spread. The frequencies of the ground roll range from 5–20 Hz. The valid signal frequencies of the field records are between 15–100 Hz.

Problems with the post-stack images

After part of coal seam 1 was taken away, the second 3D survey was performed in 2005. Figure 3 shows a migrated section in the cross-line direction by applying post-stack time migration through the mining zones. The shot records are from the second seismic data acquired in 2005. The images around the mining zones are obscure. Note that the events of coal seam 1 and seam 2 are seriously distorted. Note also that the fault terminations between the reflections of active mine and non-mining seam 1 are blurred.
migration to the second 3D seismic data. The symbols W1 and W2 marked on Fig. 3 represent the locations of two boreholes close to the boundaries of the mining zones, respectively. The mining zones are located between the two wells at around 0.5 s. Because the shot and receiver locations in the second survey are not the same as those in the first acquisition, the migrated sections in the second 3D data volume are not superposed with the first migrated sections. The sections on Figs 1 and 3 intersect into each other. Since the second acquisition parameters are different from the first survey, the time of the reflections in Fig. 3 do not correspond with that of Fig. 1. However, these differences should not affect geological formation comparisons. After interpreting the 3D migration volume, two major problems were found with the images of the mining zones. The first is that the structural features of coal seam 1 and 2 in the mining zones were completely changed relative to the original horizontal structures. A small uplift is presented in the middle of coal seam 1 and 2. Compared with the drilling data of the mining zones, it seems that the thickness of the collapse is less than the actual thickness. Therefore, the undulant phenomenon of coal seam reflections is not real. The second is that reflections of coal seams in mining zones are not clearly visible. Applying the post-stack time migration method to the second seismic data obtained inconsistent images with coal geological subsurface structures. Based on the 3D post-stack time migration volume, it is difficult to accurately determine the coal structure of the mining zones. Therefore, we attempt to apply the Kirchhoff PSTM method to the second 3D seismic data, so as to acquire better images of the mining zones.

Data preparation before prestack migration

In order to acquire high signal-to-noise (S/N) ratio prestack data, some post-processing such as noise suppression and 3D residual static correction have been applied prior to imaging. As seen in Fig. 2, ground roll is the main noise in the field data. The ground roll in the 3D shot records dominates the signals from near-offset to far-offset symmetrically. Therefore, in this study the ground roll is suppressed by applying the auto-adaptation filtering method to all shot gathers. Figure 4 shows the same shot gather as shown in Fig. 2. Note that the ground roll is almost invisible and reflections originally submerged by ground roll are present in Fig. 4.

Since the surface of this 3D survey area is plain, short wavelength static corrections influence the field data during data acquisition and result in some time differences among CMP gathers. The reflection events in Fig. 5, which is a stacked

Figure 4 A shot gather corresponding to Fig. 2. The ground roll originally in Fig. 2 has been removed from the field data.

Figure 5 A stacked section with static correction problems. A false fault marked by symbol F is present around 0.3 s.
Figure 6 A stacked section corresponding to Fig. 5. Application of 3D residual static correction to CMP gathers addresses the false fault problem and the S/N ratio of Fig. 6 is improved greatly.

section, are distorted by short wavelength static corrections. A false fault marked by symbol F shows up around 0.3 s. Since residual static correction is an effective method removing short wavelength static corrections, we perform iteratively the calculation of 3D residual static correction on 3D CMP gathers until the value of the residual static correction approaches zero. Figure 6 displays a stacked section after short wavelength static corrections. The false fault originally in Fig. 5 disappears and the S/N ratio of Fig. 6 is clearly higher than that of Fig. 5.

PRESTACK TIME MIGRATION PROCESSING

Velocity model building is a key step in prestack migration. For whatever migration algorithms, even mild variations in the velocity model could cause structural distortions of time-migrated images and render them inadequate for accurate geologic interpretation of subsurface structures (Cameron, Fomel and Sethian 2008). Therefore, an accurate velocity model is required to obtain a good prestack migration result. Two major migration velocity analysis approaches were developed in the 1980s. One is the wavefield extrapolation method firstly introduced by Yilmaz and Chambers (1984), which was referred to as the focusing depth analysis method by Faye and Jeannot (1986). The other, named as residual-moveout analysis in the offset domain, was proposed by Al-Yahya (1989) and later modified by Lee and Zhang (1992) and Lafond and Levander (1993). In addition, residual-moveout analysis in the angle-domain was presented by Biondi and Symes (2004). In our work, we use the tool based on residual-moveout analysis (Al-Yahya 1989) to derive a 3D RMS velocity field.

Since coal survey targets are generally shallower than that of petroleum exploration, their data sets are quite different. In general, the length of coal seismic data records is less than 2000 ms, while the length of petroleum data records is more than 5000 ms. The CMP bin size for a 3D coal survey is usually 10 m by 10 m and the bin size in data processing becomes 5 m by 5 m, while the CMP bin size for normal 3D oil land survey is often 25 m by 50 m and the bin size of final 3D data volume becomes 25 m by 25 m. According to 3D coal seismic data with a small bin size and short data record length, we define the workflow of coal seismic data processing. Figure 7 shows the workflow of 3D PSTM processing. It is described as follows:

1. Sort the CMP volume of data to common-offset gathers and use a 3D DMO velocity field as the initial migration velocity model.
2 Apply 3D Kirchhoff PSTM to the common-offset gathers and obtain common reflection point (CRP) gathers. Output the CRP gathers over a grid of 0.2 km by 0.2 km along the in-line and the cross-line directions, referred to as the CRP gathers of target lines.

3 Evaluate the quality of the 3D velocity model by checking the event flatness of the CRP gathers. If the events of CRP gathers are not flattened, go to the next step. Otherwise, skip step 4, 5 and 6 to perform steps 7 and 8.

4 Perform inverse NMO correction by using the same velocity model as in step 2.

5 Create a 3D RMS velocity model associated with the migrated data by performing the velocity analysis based on CRP gathers.

6 Update the 3D RMS velocity model by using the velocity field from step 5 and then return to step 2 and repeat migration processing.

7 Apply Kirchhoff PSTM to the whole 3D data volume by using the satisfied RMS migration velocity model determined in step 3.

8 Obtain the final images by stacking the 3D CRP gathers.

RESULTS

Figure 8 shows a Kirchhoff PSTM section through mining zones, extracted from the second 3D data volume in the cross-line direction. This section lies in the same position as shown in Fig. 3. The most prominent feature in Fig. 8 is that the reflections of the coal seams in the mining zones are generally horizontal, unlike the distorted events of coal seam 1 and seam 2 in Fig. 3. After interpreting the 3D prestack migration volume, the normal fault planes marked by symbols $F_1$ and $F_2$, are illustrated in Fig. 8. The region between fault planes $F_1$ and $F_2$ represents the mining zones. The time differences between the active mine and non-mining seam 1 show that the thickness of the collapse ranges from 8–12 m. According to the borehole data, the actual collapse thickness is less than 5 m, which is normal. Since the collapse of the mine, loose, seismic waves propagate more slowly than before. The reflections of the collapse cause delay. The collapse thickness based on the time differences should be greater than that of the real one. On the contrary, the images through the collapse areas do not show the delay in Fig. 3, especially the uplift event of coal seam 1 in the middle. The collapse thickness is less than the real one. The images are inconsistent with the collapse.

Another apparent improvement for the prestack images is the reflections of coal seam 2 (Fig. 8), which are clearly more continuous than shown in Fig. 3.

There are also differences in the in-line direction between the images based on post-stack time migration and prestack time migration. Figure 9 shows a migrated section in the in-line direction extracted from the 3D post-stack migration volume. Figure 10 gives a migrated section extracted from the 3D Kirchhoff PSTM volume with the same position as shown in Fig. 9. Their raw data are both from the second 3D seismic survey. Note that the reflection energy of coal seam 1, marked by a red line in Fig. 9, is relatively weak between 450–650 m, while in Fig. 10, it becomes sharper, with clearly visible
Figure 10 A migrated section in the in-line direction generated by applying Kirchhoff PSTM to the second 3D seismic data. This section is located in the same position as that of Fig. 9. Stronger reflections of coal seam 1 between 450–650 m are clearly presented in Fig. 10.

Figure 11 Partial 3D prestack migrated data volume generated by removing the layers above coal seam 1 from the whole 3D data volumes. The mining zones clearly appear on the map.

reflections of coal seam 1. Therefore, the images based on prestack time migration in the in-line direction are also improved in comparison with the images using post-stack time migration.

Figure 11 illustrates the 3D imaging map acquired by using 3D Kirchhoff PSTM, except that the reflections above coal seam 1 were removed from the whole 3D migrated data volume. A groove at about 5 m depth in the vertical direction is presented on the map. The groove shape represents the subsurface mining zones, which is consistent with the subsurface coal configuration.

In comparison with post-stack time migration in the in-line and the cross-line directions, the 3D Kirchhoff PSTM process results in much better images, which are consistent with actual coal seam structures. Therefore, the 3D PSTM volume provides relatively reliable information for analysing the coal structure of mining zones.

CONCLUSIONS

Since post-stack time migration failed to obtain correct images of coal seismic data from mining zones, we present a strategy where prestack Kirchhoff time migration was implemented on coal seismic data. Through the construction of a 3D RMS velocity field and Kirchhoff PSTM, reliable images of 3D coal seismic data are produced. We illustrate the effect of 3D Kirchhoff PSTM in the in-line and cross-line directions, respectively. Comparison with post-stack time migration displaying the images of coal seams around mining zones based on Kirchhoff PSTM are more consistent with the subsurface structures and the collapse of the mine in the Kirchhoff PSTM sections is more clear. The examples show that the Kirchhoff PSTM method can be used in the imaging of coal seismic data from mining zones. Like the imaging of petroleum seismic data, many factors also influence the quality of the images in coal PSTM processing. Besides noise suppression and 3D residual static correction guaranteeing the high S/N ratio data sets for migration, the key to the 3D Kirchhoff PSTM process is to build an accurate 3D velocity model, since a 3D velocity model is the key parameter to PSTM. In addition, since coal seams are usually shallower than oil layers, more attention should be paid to determining muting parameters in seismic data.

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REFERENCES


APPENDIX A

Differentiating $R = \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}$, we obtain the following equation:

$$\frac{\partial}{\partial z_0} \delta \left( t - t_0 - \frac{R}{V} \right) = - \frac{R}{V} \frac{\partial}{\partial z} \delta \left( t - t_0 - \frac{R}{V} \right).$$

(A1)

Upon substituting equation (A1) into integral equation (7), we have

$$P(x, y, z, t) = 
\frac{1}{2\pi} \frac{\partial}{\partial z} \int d\theta_0 \int dA_0 \frac{P(x_0, y_0, z_0, \theta_0) \delta \left( t - t_0 - \frac{R}{V} \right)}{R}$$

$$= - \frac{1}{2\pi} \frac{\partial}{\partial z} \int dA_0 \int d\theta_0 \frac{P(x_0, y_0, z_0, \theta_0) \delta \left( t - t_0 - \frac{R}{V} \right)}{R}$$

(A2)

Supposing $P(x, y, 0, t)$ is the one-way wavefield observed on $A_0$, we derive the function of the wavefield at the subsurface depth $z$ through downward extrapolation to the plane $z$.

Equation (A2) will be written as the form,

$$P(x, y, z, t) = 
\frac{1}{2\pi} \frac{\partial}{\partial z} \int dA_0 \frac{P(x_0, y_0, 0, t + \frac{R}{V})}{R}.$$  

(A3)

Equation (A3) is the far-field approximation of the integral for the homogeneous case.