Fractal models for estimating local reserves with different mineralization qualities and spatial variations

Qingfei Wang, Jun Deng, Huan Liu, Yanru Wang, Xiang Sun, Li Wan

Abstract

The concentration-area model, one of the widely applied fractal models, is utilized to describe the spatial distribution of mineralization variables, i.e., orebody thickness and grade-thickness. And based on the concentration-area model, a new fractal model for reserve estimation (abbreviated as FMRE-CA) is established. Via the demarcation values obtained in the concentration-area model, the orebody is spatially divided into several parts with different value ranges and spatial variation of mineralization variable. Based on the FMRE-CA, the local ore reserves in the parts are estimated, and the global reserve is obtained by addition of the local reserves. In the FMRE-CA, the spatial variation of the mineralization variable in a local reserve is characterized by a fractal dimension, and a greater fractal dimension denotes a more variation. Compared to traditional reserve estimations based on discrete and linear functions, the FMRE-CA is established via continuous and nonlinear function, and it is easier in calculation process and different in the way in which the local reserves are delimited. Compared to another fractal model for reserve estimation based on number-size model (abbreviated as FMRE-NS), the FMRE-CA is capable of estimating the local reserves. A gold orebody in Southwest Yunnan and two bauxite orebodies in Western Guangxi, China, are selected for case study. In the case study, the global reserve calculated via the FMRE-CA and those derived from the FMRE-NS and traditional geometric block method are analogous.

1. Introduction

In ore reserve estimation, a crucial topic in the exploration and exploitation of mineral deposits, we need not only to estimate the global ore reserve, but also to know the distribution of local reserve and ore quality. The traditional approaches for ore reserve estimation include geometric and geostatistic methods, in which the global and local reserves are estimated mostly via discrete and linear functions based on the mineralization variables, i.e., orebody thickness, grade and grade-thickness, in the exploration or mining works. For instance, in the geometric polygon method, the orebody is divided into several polygons with the exploration works as the vertexes; and the local reserve in the polygon is obtained by multiplying the polygon area and the average ore grade-thickness or orebody thickness in the polygon vertexes. In the geostatistic method, the orebody is divided into a matrix of rectangular blocks, normally with side of one-half to one-fourth the average spacing of exploration works, and the grade of each block is estimated by searching the database for the variables surrounding each block and computing the weighted average of those variables via kriging method (Annels, 1991). It is shown that, in the traditional methods, the local ore reserve is mostly determined by the locations of exploration works rather than the mineralization qualities, represented by the size of mineralization variables. Because of the complex ore-forming situation and process, the mineralization variables in an orebody often display irregular and heterogeneous spatial distributions. The spatial mineralization qualities and their spatial variations are important for ore mining and ore genesis understanding; however they are only roughly analyzed in the traditional methods. In addition, the traditional methods are often based on linear mathematics and difficult for dealing with the irregular and skewed distributions of mineralization variables (Bárdossy et al., 2003; Wang et al., 2010a), and also require relatively complex data processing. A new easy approach of partitioning the orebody into several parts according to the mineralization intensity and spatial variation and calculating the local reserves in the parts is meaningful and scientific for ore reserve calculation.

Since the geological objects commonly show irregular, heterogeneous, and skewed characteristics, they have been described by various fractal models, including box counting model (Deng et al., 2001, 2006, 2008a; Mandelbrot, 1983), number-size model (Deng et al., 2009, 2010a; Mandelbrot, 1983; Turcotte, 2002; Wan et al., 2010; Wang et al., 2010a, 2010c; Zuo et al., 2008), concentration-area model (Cheng et al., 1994),...
self-affine model (Deng et al., 2008b; Wang et al., 2007; Zuo et al., 2009), and multifractal model (Agterberg et al., 1996; Wang et al., 2008). The number-size model was widely applied to characterize the distribution of the various geological objects (Clark et al., 1995; Mandelbrot, 1983; Manning, 1994; Roberts et al., 1998; Sanderson et al., 1994; Turcotte, 2002; Walsh et al., 1991; Watterson et al., 1996; Yielding et al., 1996). Wang et al. (2010a) used the number-size model to describe the mineralization variables in the exploration works in a single deposit, and then established a fractal model for estimating global ore reserve (abbreviated as FMRE-NS) based on the number-size model. The FMRE-NS is easier in calculation process and more effective to deal with the irregularity and skewness than the geometric and geostatistical methods. However, the number-size model is not concerned with the spatial locations of mineralization variables, and thus the FMRE-NS cannot estimate the local reserve.

The concentration-area model was proposed by Cheng et al. (1994) and commonly utilized to deal with the spatial structure of geochemical map and determine the thresholds for anomalies (Cheng, 1999). The concentration-area model is expert in spatial analysis of data, which is absent in the number-size model and FMRE-NS. In this paper, the concentration-area model is utilized to describe the spatial distributions of mineralization variables in a single orebody; and according to the model, the orebody is divided into several parts with different mineralization qualities and spatial variations. Furthermore, based on the concentration-area model, we deduce another fractal model for reserve estimation, abbreviated as FMRE-CA. The local reserves with different mineralization distributions and the global reserve in the single orebody can be estimated via FMRE-CA. One gold orebody in the Yunnan province and two bauxite orebodies in the Guangxi province, China, are selected for case study.

2. Mathematical modeling

2.1. Preliminary processing of raw data

The preliminary processing of raw data in FMRE-CA is the same with the geometric block method (abbreviated as GBM) and FMRE-NS (Wang et al., 2010a). The FMRE-CA is based on the mineralization variables in the exploration works in an orebody. The orebody thickness and grade-thickness are used for estimating ore tonnage and metal tonnage, separately. The establishment and application of FMRE-CA are performed in a VLP (a vertical longitudinal projection, i.e. project each exploration work horizontally onto a vertical plane) if the orebody dip is no less than 45° or in a HLP (horizontal longitudinal projection, i.e. project each exploration work vertically onto a horizontal plane) if the dip is smaller than 45°. The mathematical modeling in the VLP is the same with that in the HLP. The following modeling is performed in a VLP.

In a VLP, the orebody horizontal thickness, average grade and grade-thickness in each exploration work are calculated. According to the cutoff and minimal mining thickness, the orebody outlines are first delimited and the orebody area is obtained. Based on the discrete exploration works, the contour maps of mineralization variables are obtained via inverse distance weighting (IDW) interpolation, which is one of the most commonly used techniques for interpolation of scatter points and widely applied in the mineral evaluation and concentration-area model (Annels, 1991; Cheng et al., 1994; Deng et al., 2010a). The IDW method is based on the assumption that the interpolating point should be influenced most by the nearby variables and less by the more distant ones. The interpolating surface is a weighted average of the scatter points in a given search area and the weight assigned to each scatter point diminishes as the distance from the interpolation point to the scatter point increases. In the application of IDW interpolation, it uses a moving circular window with adjustable parameters to control the weighting of values at neighboring points. The parameters include radius of the circular window, the weighting power, the maximum number of samples to be included within each window and the interpolation interval. The contour map used in the concentration-area model is selected as a grid format in this paper.

2.2. Concentration-area model

In the concentration-area model applied in geochemical analysis, the area \(A(\geq r)\) enclosed by contours with a concentration \(r\) has a power-law relationship with the \(r\) as follows (Cheng et al., 1994):

\[
A(\geq r) = C r^{-D}
\]

where \(D\) is called fractal dimension, \(C\) is a constant. In this paper, the concentration-area model is applied to analyze the contour maps of mineralization variables; and \(r\) represents the orebody thickness \(t_i\) or grade-thickness \(l_i\).

If the plots of \(A(\geq r)\) and \(r\) in \(ln-ln\) coordinates can be fitted with one straight line, the distribution is called a simple fractal; yet in most cases, the plots are fitted with several straight line segments, the model is called a bifractal. In a bifractal model, the variable values are divided into several segments, with demarcation value \(R_i\) (i.e. starting point), dimension \(D_i\) and constant \(C_i\) for the \(i\)th segment. A demarcation value \(R_i\) is a value of \(r\) that divides two adjoining straight line segments. The \(R_i\) is expressed as \(T_i\) and \(L_i\) for orebody thickness and grade-thickness distribution, respectively.

In the concentration-area model, the fractal dimension can indicate the spatial distribution pattern of the mineralization variable. Greater fractal dimension suggests a faster decrease of orebody thickness or density contours around the value, denoting a greater spatial variation.

2.3. Deduction of FMRE-CA

Assuming the concentration-area model comprises \(n\) straight segments, the \(i\)th segment is confined by \(R_i\) and \(R_{i+1}\) \((i = 1, 2, \ldots, n)\), \(R_1\) is the minimum value of the variable and \(R_{n+1}\) is the maximum value). According to the demarcation values, the contour map is spatially divided into \(n\) parts and the mineralization qualities are separated into \(n\) ranks; the variables in \(i\)th part range from \(R_i\) and \(R_{i+1}\) and their distribution is characterized by a fractal dimension \(D_i\).

By assuming orebody thickness is a continuous variable, the local ore tonnage \(O_i\) in the \(i\)th part can be expressed as:

\[
O_i = \int_{R_{i+1}}^{R_{i}} \frac{dA(\geq r)}{dr} dr = \frac{\rho C_i D_i}{1-D_i} \left[ T_i^{1-D_i} - T_{i+1}^{1-D_i} \right] \quad (D\neq 1)
\]

where \(\rho\) is the average ore density.

Then, the global ore tonnage \(O\) can be written as:

\[
O = \sum_{i=1}^{n} O_i = \sum_{i=1}^{n} \left[ \frac{\rho C_i D_i}{1-D_i} \left( T_i^{1-D_i} - T_{i+1}^{1-D_i} \right) \right] \quad (D\neq 1)
\]

Similarly, the local metal tonnage \(M_i\) can be obtained and expressed as:

\[
M_i = \frac{\rho C_i D_i}{1-D_i} \left[ L_i^{1-D_i} - L_{i+1}^{1-D_i} \right] \quad (D\neq 1)
\]

Then the global metal tonnage is:

\[
M = \sum_{i=1}^{n} \left[ \frac{\rho C_i D_i}{1-D_i} \left( L_i^{1-D_i} - L_{i+1}^{1-D_i} \right) \right] \quad (D\neq 1)
\]

The average grade \(G\) in the orebody is estimated by the following Eq. (6):

\[
G = \frac{M}{O}
\]
3. Applications and advantages of the FMRE-CA

3.1. Model application

Just like GBM and FMRE-NS, the FMRE-CA is only based on the orebody thickness and grade-thickness in exploration works, and has no relationship with the genetic type of deposits. Therefore, the FMRE-CA can be widely applied to the deposits of various genetic types with the precondition of that the orebody mineralization variables can be described by the concentration-area model. In the FMRE, mineralization variables are considered to be continuous, indicating that the orebody area is filled with exploration works.
infinitely small intervals, and thus the estimated results denote actual reserves which can be mined out without consideration of ore loss rate in the orebody.

In FMRE-CA, it is presumed that the data distribution obtained from a sparse grid is analogous to that derived from a dense grid. However, it is necessary to note that the denser exploration grid can provide more accurate fractal model and orebody area, and thus better estimation results. In addition, the ore density is considered to be a constant in the model; therefore, the reserves of ores of totally different industrial types or densities should be estimated separately.

The following four steps are required in the FMRE-CA: (1) delimiting orebody area in a VLP or HLP; (2) making the contour maps of the orebody thickness and grade-thickness via IDW interpolation; (3) applying the concentration-area model to the contour maps to determine the demarcation values, and dividing the orebody into several parts with various ore quality ranks according to the demarcation values; (4) calculating the local reserves with various ore quality ranks and the global reserve of the orebody.

3.2. Comparison with traditional methods

In the traditional methods, the ore blocks are partitioned according to the exploration grid, and the local reserve is estimated via discrete and linear function. However, in the FMRE-CA, the local reserves are divided in terms of the intrinsic orebody features, i.e., mineralization qualities and spatial variations. In each local reserve, the orebody area in which the mineralization variable is no less than a given value shows a fractal (power-law) relationship with the value, and this relationship is characterized by a fractal dimension; and the greater fractal dimension means the orebody area decreases more rapidly as the mineralization variable increases, suggesting a more varied spatial change of the mineralization quality. Furthermore, since the FMRE-CA is established via nonlinear mathematics and thus is much effective in fitting the skewed distribution, which is an obstacle for the traditional geometric and geostatistic methods. Therefore, FMRE-CA is considered to obtain more accurate result than the traditional methods.

3.3. Comparison with FMRE-NS

The mineralization variables are assumed to conform to the number-size model proposed by Mandelbrot (1983):

\[ N(\geq r) = C r^{-D} \]  

(7)

where \( N(\geq r) \) stands for the cumulative number of exploration works in which the variable is not less than \( r \). For bifractals, the global ore tonnage and global metal tonnage in the FMRE-NS can be expressed by Eqs. (8) and (9), respectively:

\[ O = \rho A T^D_1 \left[ D_1 \frac{T_1^{1-D_1} - T_1^{1-D_i}}{(1-D_1)} + \sum_{i=2}^{n} D_i \frac{D_i^{1-D_1} - D_i^{1-D_i}}{(1-D_i)} \right] \]  

(8)

\[ M = \rho A L^D_1 \left[ D_1 \frac{L_1^{1-D_1} - L_1^{1-D_i}}{(1-D_1)} + \sum_{i=2}^{n} D_i \frac{D_i^{1-D_1} - D_i^{1-D_i}}{(1-D_i)} \right] \]  

(9)

![Fig. 3. Locations of exploration works, orebody grade-thickness contour map and the divisions of the different parts for local reserve estimation in the orebody 1. (a) Contour map of orebody grade-thickness showing the locations of exploration works, (b) partitioned parts for local reserve estimation.](image)
Comparison to FMRE-NS, the FMRE-CA can deal with the spatial distribution of the ore reserves and estimate the local reserves with various ore quality ranks.

4. Case study 1

4.1. Deposit geology and raw data

A medium-size gold deposit in the Southwest Yunnan, China is selected for case study 1 (Fig. 1). The ores are mainly hosted in the sericite-quartz schist of the Manlai formation in the Late Proterozoic Lancang Group, clastic rocks of the Middle Jurassic Huakaizuo formation and Quaternary lateritic. Most orebodies have the characteristics of hot spring-type mineralization, while small part suffered weathering and transformed into the lateritic-type. The wallrock alterations mainly include silification, clayization, sericitization, and pyritization. The ore minerals comprise quartz, albite, plagioclase, kaolinite, mica, sericite, pyrite and limonite.

Orebody 1 in the gold deposit is selected for reserve estimation and the data of exploration works are from Yunnan Bureau of Geology & Mineral Resources. In orebody 1, total 77 exploration works including both drills and wells are carried out and their locations are shown in Figs. 2 and 3. In the exploration, the orebody area, ore density, ore tonnage, metal tonnage and orebody grade are estimated via the GBM to be 263,149.33 m², 1.75 t/m³, 5.91 Mt, 4.59 t and 0.78 g/t, respectively.

The histograms of the natural logarithms of thickness and grade-thickness in exploration works in the orebody 1 are illustrated in Fig. 4a and c, and the ‘quantile–quantile’ plots (Q–Q plot) in Fig. 4b and d. The Q–Q plots display that the variables do not follow log-normal distributions, especially in the two tails; the variables have skewed distributions. The mineralization variables in orebody 1 have much varied spatial changes as shown in Fig. 5.

4.2. FMRE-CA application

Since the orebody is nearly horizontal, the HLP is applicable. The parameters in the IDW interpolation were set as: radius of 400 m, weighting power of 2 (corresponding to a linear weighting function with weight 0 for the points located at the boundary or outside the moving window, and 1 for the point situated at the center), maximum of 15 samples per window (if more than 15 samples occur within the window for a given location, only the 15 nearest points are selected) and interpolation interval of 15 m. The contour maps of thickness and grade-thickness in orebody 1 are illustrated in Figs. 2a and 3a, respectively.

After the contour maps of the thickness and grade-thickness are obtained, they are analyzed via concentration-area model. The concentration-area model for thickness contour map comprises 6 segments, as is illustrated in Fig. 6a, the fractal dimensions and demarcation values for the 6 segments are listed in Table 1. According to the demarcation values, the orebody thickness is divided into 6 ranks, and the thickness contour map is correspondingly separated into 6 parts as shown in Fig. 2b. Generally, with the raise of mineralization rank, the fractal dimension increases gradually, suggesting an upgrade of spatial variation. The local ore tonnage in each part is calculated in light of Eq. (2) and the results are listed in Table 1, and the estimated global ore tonnage is 6.08 Mt. Percentages of the local ore tonnages with various mineralization qualities are shown in Fig. 7a, and it is revealed that the local ore tonnage with mineralization quality rank III takes up 57.44% of the global reserve.

The concentration-area model for grade-thickness is composed of 7 segments (Fig. 6b), and correspondingly, the orebody area is partitioned into 7 parts (Fig. 6b). As is similar to orebody thickness distribution, the spatial variation of grade-thickness grows approximately as the mineralization rank rises. The percentages of local metal tonnages in the 7 parts calculated via Eq. (4) are shown in Fig. 7b, and
Fig. 5. Orebody thickness and grade-thickness curves along sections in the orebody 1. (a) Thickness curve along section AA'; (b) grade-thickness curve along section AA'; (c) thickness curve along section BB'; (d) grade-thickness curve along section BB'.

Fig. 6. Concentration-area models for orebody thickness and grade-thickness distributions in the orebody 1. The different shades of gray represent the different size segments. (a) Concentration-area model of orebody thickness, (b) concentration-area model of orebody grade-thickness.

Table 1
Fractal parameters in the concentration-area model for thickness distribution in orebody 1 and the reserves with different ore qualities estimated via FMRE-CA.

<table>
<thead>
<tr>
<th>Ore quality rank</th>
<th>Orebody area $(m^2)$</th>
<th>Ore density $(t/m^3)$</th>
<th>Fractal parameters</th>
<th>Local ore tonnage $(Mt)$</th>
<th>Global ore tonnage $(Mt)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>24,356.94</td>
<td>1.75</td>
<td>$r_i = 0.09x+12.58$, $R^2 = 0.72$</td>
<td>$y_i = -1.22x+14.89$, $R^2 = 0.99$</td>
<td>$C_i = 0.09x+12.58$, $R^2 = 0.72$</td>
</tr>
<tr>
<td>II</td>
<td>83,105.00</td>
<td>1.75</td>
<td>$r_i = 0.8x+14.89$, $R^2 = 0.99$</td>
<td>$y_i = -1.74x+15.70$, $R^2 = 0.74$</td>
<td>$C_i = 0.8x+14.89$, $R^2 = 0.99$</td>
</tr>
<tr>
<td>III</td>
<td>133,921.30</td>
<td>1.75</td>
<td>$r_i = 0.8x+14.89$, $R^2 = 0.99$</td>
<td>$y_i = -3.63x+20.27$, $R^2 = 1.00$</td>
<td>$C_i = 0.8x+14.89$, $R^2 = 0.99$</td>
</tr>
<tr>
<td>IV</td>
<td>14,192.59</td>
<td>1.75</td>
<td>$r_i = 0.8x+14.89$, $R^2 = 0.99$</td>
<td>$y_i = -8.80x+36.87$, $R^2 = 1.00$</td>
<td>$C_i = 0.8x+14.89$, $R^2 = 0.99$</td>
</tr>
<tr>
<td>V</td>
<td>71,751.12</td>
<td>1.75</td>
<td>$r_i = 0.8x+14.89$, $R^2 = 0.99$</td>
<td>$y_i = -35.43x+129.99$, $R^2 = 1.00$</td>
<td>$C_i = 0.8x+14.89$, $R^2 = 0.99$</td>
</tr>
<tr>
<td>VI</td>
<td>400.00</td>
<td>1.75</td>
<td>$r_i = 0.8x+14.89$, $R^2 = 0.99$</td>
<td>$y_i = -18.29x+71.87$, $R^2 = 0.98$</td>
<td>$C_i = 0.8x+14.89$, $R^2 = 0.99$</td>
</tr>
</tbody>
</table>
the metal tonnage with mineralization quality rank IV occupies 43.1% of the global reserve. Based on Eq. (5), the global metal tonnage is estimated to be 4.80 t (Table 2).

4.3. Estimation result comparisons

The number-size models of thickness and grade-thickness distributions in the orebody 1 are illustrated in Fig. 8a and b, respectively. According to the number-size models and based on Eqs. (8) and (9), the estimated global ore tonnage and metal tonnage in the orebody 1 are 6.37 Mt and 5.10 t respectively (Table 3). The global reserves and ore grade obtained via FMRE-CA, GBM and FMRE-NS are compared, and it is shown that the FMRE-CA has similar results to the other two methods (Fig. 9). The relative errors between the reserve estimated by FMRE-CA and that calculated via GBM are 2.85% for ore tonnage, 4.76% for metal tonnage and 1.25% for ore grade; in addition, those between the FMRE-CA and FMRE-NS are 4.57% for ore tonnage, 5.83% for metal tonnage and 1.32% for ore grade.

Table 2
Fractal parameters in the concentration-area model for grade-thickness distribution in orebody 1 and the reserves with different ore qualities estimated via FMRE-CA.

<table>
<thead>
<tr>
<th>Ore quality rank</th>
<th>Orebody area (m²)</th>
<th>Ore density (t/m³)</th>
<th>Fractal parameters</th>
<th>Local metal tonnage (t)</th>
<th>Global metal tonnage (t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>26,990.65</td>
<td>1.75</td>
<td>1.35</td>
<td>0.09</td>
<td>5.89</td>
</tr>
<tr>
<td>II</td>
<td>57,018.56</td>
<td>1.75</td>
<td>5.89</td>
<td>0.96</td>
<td>7.90</td>
</tr>
<tr>
<td>III</td>
<td>78,807.10</td>
<td>1.75</td>
<td>7.90</td>
<td>1.74</td>
<td>10.98</td>
</tr>
<tr>
<td>IV</td>
<td>85,622.04</td>
<td>1.75</td>
<td>10.98</td>
<td>3.63</td>
<td>18.20</td>
</tr>
<tr>
<td>V</td>
<td>10,012.61</td>
<td>1.75</td>
<td>18.20</td>
<td>3.36</td>
<td>25.35</td>
</tr>
<tr>
<td>VI</td>
<td>4300.00</td>
<td>1.75</td>
<td>25.35</td>
<td>8.80</td>
<td>33.02</td>
</tr>
<tr>
<td>VII</td>
<td>400.00</td>
<td>1.75</td>
<td>33.02</td>
<td>35.43</td>
<td>34.18</td>
</tr>
</tbody>
</table>

Fig. 7. Pie diagrams of local reserves with different mineralization qualities and spatial variations in the orebody 1. (a) Ore tonnage; (b) metal tonnage.

Fig. 8. Number-size models for the thickness and grade-thickness distributions in the orebody 1. The different shades of gray represent the different size segments. (a) Number-size model of orebody thickness, (b) number-size model of orebody grade-thickness.
5. Case study 2

5.1. Deposit geology and raw data

Two typical orebodies in the Xinxu bauxite deposit, Western Guangxi, China are chosen for case study 2. The deposit strata from oldest to youngest include Devonian carbonates and shale, Carboniferous carbonates, Permian limestone, siliceous limestone and bauxite ores, Triassic sandstone and carbonates and Quaternary laterite (Fig. 10). The bauxite orebodies with thickness from 1.22 m to 25.3 m are contained in the Quaternary karst depressions, which are widely developed in the deposit. The Quaternary bauxite ores were transformed from the Permian bauxite ores during the formation of Quaternary karst terrain (Deng et al., 2010b). The Quaternary ores have various colors in surface, such as red, brownish red and dark red, and are mainly characterized by nodular, pseudoporphyritic and ooidic textures. The ores are mainly composed of diasporite, hematite, goethite, gibbsite, amesite, kaolinite, chamotte and small amount of debris minerals, e.g., rutile and zircon (Liu et al., 2010; Wang et al., 2010b).

In the Xinxu deposit, orebody 2 and orebody 3 are selected for reserve estimation and the exploration data are from bureau of geoexploration and mineral development of Guangxi. Total 87 wells are evenly distributed in the orebody 2, and 35 wells in orebody 3, as shown in Figs. 11 and 12. Since the orebodies are horizontal, the HLP is applied; and vertical orebody thickness is used for ore tonnage estimation. Via the GBM, the orebody area, ore density and global ore tonnage in orebody 2 are calculated to be 155,509 m², 784 kg/m³ and 1.78 Mt, respectively; those in orebody 3 are 56,259 m², 1054 kg/m³ and 0.56 Mt. The histograms of the natural logarithms of orebody thickness in the orebody 2 and orebody 3 are illustrated in Fig. 13a and c, and the 'quantile–quantile' plots (Q–Q plot) in Fig. 13b and d, respectively. The Q–Q plots show that the thickness distributions are skewed. It is further shown in the Fig. 14 that the spatial changes of orebody thickness are irregular in the two orebodies.

5.2. FMRE-CA application

The contour maps of the orebody thickness in the orebody 2 and orebody 3 are produced via IDW method and illustrated in Figs. 11a and 12a, respectively. The parameters involved in the IDW method are set as: radius of 270 m, weighting power of 2, maximum of 10 samples per window and interpolation interval of 10 m. Based on the thickness contour maps, the concentration-area models have been established. The concentration-area models in the orebodies 2 and 3 are both

<table>
<thead>
<tr>
<th>Orebody area (m²)</th>
<th>Density (t/m³)</th>
<th>Fractal parameters</th>
<th>Result G (g/t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>O (Mt) 2.63 x 10³</td>
<td>1.75</td>
<td>1.80 0.06 4.35 0.63 13.62 1.07 16.88 2.63 29.35 5.32 44.58 6.37 0.80</td>
<td></td>
</tr>
<tr>
<td>M (t) 1.20</td>
<td>0.14 3.60 0.51 11.89 1.73 28.14 3.73 37.89 5.10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
composed of 5 segments (Fig. 15). According to the demarcation values in the concentration-area models, the thickness maps of orebodies 2 and 3 are both divided into 5 parts as shown in Figs. 11b and 12b separately. Compared to the orebody 1, the fractal dimension in the models of the orebodies 2 and 3 changes more abruptly as the mineralization quality rank increase. It is shown that the fractal dimensions in segments 4 and 5 in the concentration-area models are much greater than those in segments 1 and 2 in the two orebodies, suggesting the orebody thickness in the parts with ranks 4 and 5 have much more variation. Based on Eq. (3), the local ore tonnages in each part in orebody 2 and orebody 3 are calculated and listed in Table 4, and percentages of the local ore tonnage in orebody 2 and orebody 3 are illustrated in Fig. 16a and b, respectively. Finally, the global ore tonnage in orebody 2 is estimated to be 0.551 Mt, and that in orebody 3 is 1.869 Mt. The local reserve with mineralization quality rank IV is the greatest in the five local reserves in both orebody 2 and orebody 3, and it accounts for 42.48% of the global reserve in orebody 2, and it occupies 37.57% in orebody 3.
5.3. Estimation result comparisons

The number-size models of thickness distribution in the orebody 2 and orebody 3 are illustrated in Fig. 17a and b separately. According to number-size models and Eq. (7), the global reserves in the two orebodies are estimated (Table 5). Comparing the global reserves calculated via FMRE-CA, GBM and FMRE-NS, it reveals that the FMRE-CA result is roughly equivalent to those of the other two methods (Fig. 18). The relative errors between the global reserve derived from the FMRE-CA and that calculated by the GBM are 4.84% for orebody 2 and 1.24% for orebody 3; and those between the FMRE-CA and FMRE-NS are 6.73% for orebody 2 and 3.91% for the other.

6. Conclusions

In this paper, a new fractal model for reserve estimation based on the concentration-area model is established. Compared to the FMRE-NS, the FMRE-CA analyzes the spatial distribution of the ore reserves. Compared
to the traditional geostatistical and geometric methods, the FMRE-CA is established based on continuous and nonlinear function, and it is not only easier in calculation process, but also prominent in delimitating the local reserves in terms of the different size ranges and spatial variation of the mineralization variables. In the FMRE-CA, the variable distribution in a local reserve is characterized by a size range and a fractal dimension, and a greater fractal dimension denotes a more varied spatial change of the mineralization quality. One gold orebody and two bauxite orebodies in China were used in the case studies. It is revealed that the estimated global reserves via FMRE-CA and those derived from traditional GBM and from FMRE-NS are similar, with relative errors less than 6.73% and 5.83% for global ore tonnage and metal tonnage respectively. The case studies further approve that the FMRE-CA can be utilized in the deposits with various genetic types.

<table>
<thead>
<tr>
<th>Orebody 2</th>
<th>Orebody area (m²)</th>
<th>Density (kg/m³)</th>
<th>Fractal parameters</th>
<th>Local ore tonnage (Mt)</th>
<th>Global ore tonnage (Mt)</th>
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<td>I</td>
<td>7084.93</td>
<td>784</td>
<td>1.62</td>
<td>0.03</td>
<td>1.61 x 10⁵</td>
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<td>11.15</td>
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</tr>
<tr>
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<td>13.88</td>
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<td>15.50</td>
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<tr>
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<td>V</td>
<td>900.00</td>
<td>784</td>
<td>22.42</td>
<td>19.59</td>
<td>25.09</td>
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<tr>
<td>Orebody 3</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>1054</td>
<td>14.43</td>
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Fig. 15. Concentration-area models for the thickness distributions in the orebodies 2 and 3 in the Xinxu deposit, China. The different shades of gray represent the different size segments. (a) orebody 2, (b) orebody 3.

Fig. 16. Pie diagrams of local ore tonnages with different mineralization qualities and spatial variations in orebodies 2 and 3 in the Xinxu deposit, China. (a) Orebody 2; (b) orebody 3.
Fig. 18. Comparisons of global ore tonnages and ore grades obtained by the FMRE-CA, FMRE-NS and GMB for the orebody 2 and orebody 3 in the Xinxu deposit, China.

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References


