Simultaneous rejuvenation and aging of groundwater in basins due to depth-decaying hydraulic conductivity and porosity

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The age of groundwater is a manifestation of the temporal scale of groundwater flow in basins, whose pattern was recently found to be influenced by depth-dependent hydraulic conductivity (K). In this paper, we show through numerical simulations how well-documented depth-decaying K and porosity (θ) influence groundwater age. In the unit basin, depth-decaying K and θ cause aging in deeper parts and rejuvenation near the discharge zones, and the size of rejuvenated zones decreases with the decay exponent (A). In the Tóth basin, the geometry and size of rejuvenated zones, which are generally located at the interfaces between flow systems in the mid to lower reaches of the basin, are sensitive to A. In both basins, the maximum relative age and the relative age of groundwater at the lowest discharge point are dependent on A. Therefore, the depth-decaying K and θ cannot be ignored when interpreting groundwater age distribution.

1. Introduction

Groundwater is a ubiquitous geologic agent [Tóth, 2009] and its age and residence time determines its impact on geologic processes [Garven, 1995; Alley et al., 2002; Ingebritsen et al., 2006] and also controls the feedbacks of these processes on groundwater flow and transport [Bethke and Johnson, 2002, 2008]. However, determination of groundwater age is complicated by the fact that there is no single age for a groundwater parcel. The apparent age of groundwater is a result of mixing and transport processes of different fluid parcels. This is best illustrated by the concept of “age mass”, the product of the water mass and its age. Goode [1996] presented the governing equation for age transport, which is analogous to that for solute transport. Therefore, the processes that control solute transport would also control groundwater age distribution in aquifers, and any progress towards understanding processes affecting groundwater age are dependent on how well flow and transport processes are represented.

2. Methods

Following Tóth [1963] and Freeze and Witherspoon [1967], the average level of water table could be assumed constant under natural equilibrium conditions and a two-dimensional cross-section of a basin is representative of a three-dimensional basin when it is taken parallel to the direction of dip of the water table slope. In our study, groundwater age distribution is modeled by numerically solving the two-dimensional steady-state groundwater flow and age transport equations for simple bounded rectangular basins but with a sinusoidally-varying top boundary representing both the ground surface and the water table, i.e., the unsaturated zone is negligible. The governing equations are solved with the finite-element method implemented in COMSOL Multiphysics. The water table which has an elevation \( z_s(x) \) corresponds to a specified head boundary condition with \( h(x, z_s) \). We consider two cases for the top boundary, one for a unit basin with:

\[
z_s(x) = z_0 + a \cos \left( \frac{2\pi x}{\lambda} \right)
\]

where \( z_0 = 1000 \) m, the amplitude of the variations \( a = 20 \) m, the wavelength of periodic topographic variation \( \lambda = 1500 \) m, and \( x \) ranges from 0 to 750 m. The basin bottom is set

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at \( z = 0 \) m. The second case follows the classic Tóth basin and has:

\[
z_s(x) = z_0 + x \tan \alpha + \frac{a}{\cos \alpha} \sin \left( \frac{2\pi x}{\lambda \cos \alpha} \right)
\]

where \( z_0 = 1000 \) m, \( \tan \alpha = 0.02 \), \( a = 15 \) m, \( \lambda = 1500 \) m, and \( x \) ranges from 0 to 6000 m. Except for the top boundary which follows (1) or (2), all other boundaries assume zero normal flux.

[5] The dependence of \( K \) on depth is represented using the exponential decay model by assuming locally isotropic conditions:

\[
K(x, z) = K_0 \exp[-A(z_s(x) - z)]
\]

where \( K_0 \) is the \( K \) at the ground surface, \( A \) is the decay exponent which indicates the decrease rate of \( K \) with depth. Although there is no unique relationship between \( K \) and porosity (\( \theta \)), in many cases they can be related using the power law in the following form [Bernabe et al., 2003]:

\[
\frac{K}{K_0} = \left( \frac{\theta}{\theta_0} \right)^n
\]

where \( \theta_0 \) is the porosity at the ground surface and \( n \) is a medium dependent coefficient. Therefore, the depth-decaying \( \theta \) can also be described by an exponential model, which is known as Athy’s law [Athy, 1930], in the following form:

\[
\theta = \theta_0 \exp \left[ -\frac{A}{n} (z_s(x) - z) \right]
\]

where \( A \) is the decay exponent which indicates the decrease rate of \( K \) with depth. In our models, \( K_0 = 1 \) m/d and \( \theta_0 = 0.3 \). Although \( n \) varies significantly, we chose \( n = 2 \), unless noted otherwise, following a theoretical capillary network model [Bernabe et al., 2003].

[6] Although age mass transport equation with varying porosity can be readily obtained by Goode [1996], all previous studies assumed uniform porosity [Goode, 1996; Bethke and Johnson, 2002, 2008], i.e., the heterogeneity of \( \theta \) is ignored as in most studies on solute transport. For the case of varying \( \theta \), the steady-state groundwater age mass transport equation can be derived from equation (10) of Goode [1996]:

\[
\nabla \cdot (\theta D \nabla \tau) - \nabla \cdot (u \theta \tau) + \theta = 0
\]

where \( \tau \) is groundwater age, \( u = [u_x, u_z] \) is the pore velocity vector, \( D \) is the dispersion coefficient tensor. In the calculation for the elements of \( D \), we assume the effective molecular diffusion coefficient \( D^* = 1.16 \times 10^{-8} \) m²/s, the longitudinal dispersivity \( \alpha_L = 6 \) m, and the transverse
smaller and compressed towards the lowest discharge point while the aging zone becomes larger and covers more of the basin. Therefore, counter-intuitively, depth-decaying $K$ and $\theta$ actually leads to rejuvenation of groundwater in some areas within the basin. As $A$ increases, groundwater age increases in some areas and decreases in other areas of the basin.

[9] Compared with the groundwater exiting the discharge boundary at $A = 0$, older water exits from the upper part of the basin when $A > 0$, while younger water flows out from the lower part when $A > 0$ (Figure 2a). Therefore, a broader distribution of age would result due to depth-decaying $K$ and $\theta$ for water in the discharge areas, which typically correspond to springs, rivers, lakes, and wetlands. We further compared the relative age of groundwater at the lowest discharge point and the maximum relative age in the unit basin at different decay exponent, $A$ (Figure 2b). It is clear that the relative age of groundwater at the lowest point decreases exponentially with $A$, and the maximum relative age in the basin increases exponentially with $A$.

4. Cause of Simultaneous Rejuvenation and Aging: $K$ Versus $\theta$

[10] Ignoring depth-decay in $\theta$, depth-decaying $K$ mostly leads to aging everywhere in the basin with the aging systematically increasing with depth (Figure 1h), which reflects the increased residence time due to pronounced deceleration with depth. The aging is owed mainly to equation (3) with some effects of the flow field. However, depth-decaying $K$ does lead to a very small zone of rejuvenation near the discharge area (very small green-colored zone in Figure 1h). The size of this zone only slightly increases with increasing $A$.

[11] On the other hand, depth-decaying $\theta$ in the absence of decaying $K$ leads to systematic rejuvenation everywhere (Figure 1i); it is mostly a manifestation of equation (5), again with some influence from the flow field. All previous models [e.g., Goode, 1996; Bethke and Johnson, 2002, 2008] assumed that $\theta$ is uniform and failed, therefore, to recognize this effect. More importantly, when the depth-variation of both $K$ and $\theta$ are considered, the pattern of aging and rejuvenation differs greatly. The combination of depth-decaying $K$ (Figure 1h) and $\theta$ (Figure 1i) leads to the normalized relative age pattern in Figure 1f. Furthermore, an increase in $n$ which leads to a weaker decay of $\theta$ would result in a smaller zone of rejuvenation (compare Figure 1f with Figure 1j where $n = 3$ but with the same $A$).

5. Age Distribution in the Tóth Basin With Hierarchically Nested Flow Systems

[12] In the Tóth basin, regional sinusoidal topography leads to nested flow systems which in turn results in distinct groundwater age zones and sharply outlined patterns of rejuvenated and aging waters (Figure 3). The solid lines in Figures 3a–3d are to illustrate the flow patterns, with each streamtube covering 5% of the total flux. For each $A$, five local age systems can be recognized, which are highly correspondent to local flow systems. Compared with deep groundwater, the groundwater in these local age systems is relatively young. The greatest depth of a local age system, which is the depth where there is an abrupt increase in relative age (see the position where color changes in

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**Figure 2.** (a) Normalized relative age ($\tau/\tau_0$) distribution along the discharge boundary of the unit basin (location represented in inset cross-section) for different decay exponents $A$. (b) Normalized relative age ($\tau/\tau_0$) for the lowest discharge point and the maximum relative age in the unit basin.

**Figure 3.** Age Distribution in the Tóth Basin With Hierarchically Nested Flow Systems.

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Figures 3a–3c), increases with the decay exponent except for the one near the regional high. This phenomenon is similar to the increase in penetration depth of local flow systems with decay exponent as demonstrated by Jiang et al. [2009b]. Following the streamtubes from the regional high to the regional low, groundwater is increasingly older. When $A$ equals $0.01 \text{ m}^{-1}$, the regional flow system fails to develop, resulting in a maximum relative age near the deep of the local age system originating from the globally highest point.

Rejuvenated or aging zones may be present in each local age system, but their distribution in the Tóth basin is more complicated than those in the unit basin (Figures 3e–3g). The rejuvenated zone, if exists, is centered around the boundaries between local and regional (or subregional) flow systems with the normalized relative age increasing away from these cores. In the lowest local age system, the rejuvenated zone spans from one end to the other end of the local age system and may encompass the entire local age system when the decay rate is small (Figure 3e). As the distance of the local age system away from the lowest discharge point increases, the rejuvenated zone becomes more vertically constrained, becoming a discontinuous lens originating from the higher end of the local age system. In the local age systems closer to globally highest point at the upper right corner, which is the main watershed divide, the rejuvenated zones finally disappeared. Therefore, aging happens in the most parts of the basin but rejuvenation only happens locally in the lower and mid reaches of the basin.

The size of the rejuvenated zones sandwiched between local and regional flow systems is highly dependent on $A$. In the lowest local age system, the relationship between the size of rejuvenated zones and $A$ is obscure, but in the second and third local age systems from the left to the right, it is apparent that rejuvenated zones decrease in size with $A$. At the same time, Figures 3e–3g show that increasing $A$ leads to more pronounced aging in the deeper parts of the basin. Therefore, drastic decay of $K$ and $\theta$ in nested regional flow systems would lead to a very heterogeneous groundwater age distribution. Compared with the base case, areas near flow system boundaries in the lower and mid reaches of the basin may host water that is younger while deep areas may have ages that are more than a hundred times older.

We further analyzed the effect of varying $A$ on the relative age of groundwater in the lowest discharge point, as well as on the maximum relative age of groundwater in the whole basin (Figure 4). The relative age of groundwater discharging in the lowest point of the basin increases with $A$ and has a maximum value when $A < 0.0008 \text{ m}^{-1}$. This implies that the contribution to age mass from regional flow exceeds that from local flow. When $A > 0.0008 \text{ m}^{-1}$, however, owing to restriction of regional flow, the relative age of groundwater at the lowest discharge point decreases exponentially with $A$. On the other hand, the maximum relative age increases exponentially with $A$, although the position of these waters varies within the basin.

6. Summary and Conclusions

Here we show through numerical flow and transport simulations how depth-decaying hydraulic conductivity and porosity, phenomena often neglected but widely observed, leads to simultaneous aging and rejuvenation of groundwater in basins with topography-driven water flow. Depth-decaying $K$ mostly leads to aging while depth-decaying $\theta$
leads to rejuvenation of groundwater at given locations in a drainage basin. Acting together, these factors cause aging in deeper parts and rejuvenation near the discharge zones in the unit basin. Moreover, both the size of the rejuvenated zones, and the relative age of groundwater at the lowest discharge point decrease with the decay exponent while the maximum relative age in the basin increases with the decay exponent. For more complex scenarios with nested local and regional flow systems, zones of relative rejuvenation, which are located at the interface between flow systems in the mid to lower reaches of the basin, may either be continuous, spanning from one end to the other end of the local age system, or lenses originating from one end of the local age system but disappearing with depth. The water with maximum relative age in the basin exists at the lowest discharge point when the decay exponent is very small, but moves to the deep part of the basin and increases exponentially with the decay exponent when it exceeds 0.0008 m⁻¹. The effects of depth-decaying hydraulic conductivity and porosity should be considered when interpreting ages and residence times of subsurface fluids and surface waters fed by discharge from regional basins such as springs, lakes, rivers and wetlands although this has not been the practice in most past studies.

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