Fractal models for ore reserve estimation

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Abstract

Traditional geometric and geostatistic methods for reserve estimation in a single deposit are difficult to use with skewed distribution mineralization variables including grade, orebody thickness and grade-thickness, a common characteristic of most deposits, and require complex data processing. It has been shown that the skewed mineralization variables can be described by the number-size model in a fractal domain. Based on the number-size model, assuming that orebody thickness and grade-thickness are continuous variables, the fractal model for reserve estimation (FMRE) in a single deposit can be established. In the FMRE, ore tonnage can be estimated given the orebody area and the fractal parameters of orebody thickness distribution and metal tonnage can be estimated based on the orebody area and the fractal parameters of grade-thickness distribution. The reserve estimated by the FMRE can denote actual ore tonnage and metal tonnage that can be mined out of the deposit. The FMRE was applied to the Dayingezhuang gold deposit in the Jiaodong gold province in China. The gold reserves via the FMRE and the traditional geometric block method are similar, with relative errors of 3.11% in ore tonnage and 0.29% in metal tonnage. Compared to traditional reserve estimation the FMRE is much easier in calculation process and is more reasonable in dealing with the skewed distribution. However, this new method fails to calculate local reserve, which can be derived via any of the traditional estimation methods.

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1. Introduction

Ore reserve estimation has crucial importance for the evaluation and exploitation of mineral deposits. Reserve estimation is based on the results of exploration, such as drillholes and exploratory drifts, located in a grid. Several variables concerning exploration works are proposed as representations for mineralization intensity and for calculating the reserve. These variables generally include the ore thickness in the exploration works and the grade over this thickness. The grade-thickness (accumulation) value, with the units m g/t (Parker, 1991), another important variable for reserve estimation, can be obtained by multiplying the grade and the corresponding thickness. The three variables are called ‘mineralization variables’ in this paper.

1.1. Traditional reserve estimation methods

Traditional reserve estimation methods can be roughly classified into two main groups: geometric and geostatistical methods (Henley, 2000). The geometric methods comprise the block methods (triangle, regular, squares, rectangular, polygonal and irregular or geometric blocks), the methods of profiles (vertical, horizontal and inclined) and the isopach method. The geometric block method (GBM) is one of the most popular methods for reserve estimation. In the GBM, each exploration work is projected horizontally onto a vertical plane (a vertical longitudinal projection, VLP) in the case of that orebody with a dip more than 45° (or project each exploration work vertically onto a horizontal plane – a horizontal longitudinal projection, abbreviated as HLP – if the dip is smaller than 45°). Orebody area can then be manually separated into numerous blocks with exploration works in vertices of the VLP plane and the inside average grade, orebody horizontal or vertical thickness and the local reserve in each block are calculated according to the exploration works. Finally, the global reserve in a deposit is obtained by summing up the local reserve in every block (Annels, 1991).

The geostatistical methods take into account the spatial autocorrelation of mineralization variables. By the method of variography, the ranges of influence of the variables can be determined in two or three dimensions. Then, the local mineralization variables can be obtained via linear interpolation, based on the exploration works in the influence range. Geostatistical calculation requires a large sample size. With a small number of exploration works (say, about 20), the calculation of variograms becomes increasingly uncertain, even impossible (Diehl, 1997). Geostatistical calculations also require suitable computer programs and a considerable mathematical background (Bárdossy and Fodor, 2004).

Reserve estimations commonly involve extremely high values of grades and thicknesses, designated as outliers, which induce a highly skewed distribution and are obstacles to fitting traditional mathematical models.
models applied in geometric and geostatistical methods to the distributions of mineralization variables. Outliers affect the precisions of estimation result greatly (Arik, 1990; Costa, 2003). The proper handling of such outliers is crucial to ore reserve estimation. Different correction methods, including simple trimming, ignoring their potential influence and keeping them in the dataset, or adjusting the outliers according to the overall trend of the cumulative distribution functions, are proposed for dealing with the grade outliers. And few methods involving complex data processing are proposed for the skewed thickness or grade–thickness distribution (Tutmez et al., 2007). Reliable ore reserve estimates for deposits with skewed distributions of mineralization variables are still difficult to obtain when using geometric and geostatistical methods.

Based on exploration works, the geometric and geostatistical methods can estimate the discrete local reserve with linear interpolation and then calculate the global reserve in a deposit. However, both methods involve complex data processing and can present difficulties in terms of dealing with the skewed distribution of mineralization variables. In order to achieve reliable reserve estimates, a good mathematical model must be built to represent adequately the overall distribution of data.

1.2. Fractal models in geology

Natural variability is an inherent feature of geological processes and geological objects are neither random nor homogeneous; they show irregular, heterogeneous, and skewed characteristics (Bárdoossy et al., 2003). Fractal models are well-established and have been effectively applied to describe the distributions of geological objects, including the box counting model (Mandelbrot, 1983; Cheng, 1995; Deng et al., 2001, 2006), the number–size model (Mandelbrot, 1983; Turcotte, 2002; Wang et al., 2007a), the radial–density model (Mandelbrot, 1983; Feder, 1988; Carlson, 1991; Blenkinsop, 1994; Raines, 2008; Carranza, 2009), the grade–tonnage model (Turcotte, 1997, 2002; Wang et al., 2007c), the self-affine model (Wang et al., 2007b), and the multifractal model (Agterberg et al., 1996; Cheng, 1999; Deng et al., 2007, 2008a; Wang et al., 2008). Of the different fractal models, the number–size model is the most popular and it has been applied to characterize the distribution of earthquake magnitudes (Lee and Schwarz, 1995; Turcotte, 1997; Dimri, 2005), fault displacement lengths (Aviles et al., 1987; Okubo and Aki, 1987; Walsh et al., 1991; Jackson and Sanderson, 1992; Manning, 1994; Fossen and Rørnes, 1996; Knott et al., 1996; Needham et al., 1996; Nicol et al., 1996; Watterson et al., 1996; Yielding et al., 1996; Pickering et al., 1997), vein thickness and size of ore deposits (Carlson, 1991; Manning, 1994; Sanderson et al., 1994; Clark et al., 1995; McCaffrey and Johnston 1996; Roberts et al., 1998; Turcotte, 2002), and element concentrations (Monecke et al., 2001, 2005; Deng et al., 2008; Wan et al., 2010).

As the fault displacement in a fault system follows the number–size model and it can be assumed to be a continuous variable, Scholz and Cowie (1990) created the total displacement model to calculate the total displacement of a fault system; this method is easily applied to the analysis of a fault system and hydrocarbon accumulations (Walsh et al., 1991; Jackson and Sanderson, 1992; Barton and Scholz, 1995). However, because most geological objects cannot be considered continuous variables, further applications of the method proposed by Scholz and Cowie (1990) are limited. For example, because the number of deposits in an ore cluster area is limited and the ore tonnage of the deposits is a discrete variable; this method cannot be utilized to estimate regional resources.

1.3. Research objective

In an orebody, mineralization variables can be treated as continuous variables. Explorations are random samples of orebody, and data of ore thickness and grade–thickness are used for orebody modeling. We assume that the distributions of ore thickness and grade–thickness can be described by the number–size model, and then deduce the fractal models for reserve estimation, abbreviated as FMRE, via the number–size model. The Dayingezhuang deposit in the Jiaodong gold province in China is used in the case study.

2. Mathematical modeling

2.1. Preliminary processing of raw data

In the exploration of a deposit, a series of exploration works are performed along different exploration lines at various depths. Along exploration works, channel samples with constant length are completed and their metal concentrations are analyzed. Outliers in metal concentration data are replaced by the average grade of an orebody. According to the concentration distributions, cutoff grade and minimal mining thickness, the outlines of an orebody can be delimited, and the orebody thickness and the corresponding mean concentration in each exploration work are calculated. Exploration works with grades greater than cutoff and orebody thickness greater than the minimum mining thickness are categorized as “mineralized” in this paper, and their total number is \( N \), otherwise they are categorized as “non-mineralized”.

As with the traditional GBM, if an orebody dip is more than 45°, we project each exploration work horizontally onto a VLP (Fig. 1). In a VLP, the horizontal thickness of an orebody and the average grade in each exploration work are calculated, and the corresponding grade–thickness can thus be obtained. The following modeling process is carried out based on the VLP of exploration works. If an orebody dip is less than 45°, we can project each exploration work vertically onto a HLP, and orebody vertical thickness and grade–thickness in every exploration work can be obtained. In this case the mathematical modeling explained below is still applicable.
2.2. Block modeling

It can be preliminarily assumed that exploration works are evenly distributed with constant horizontal interval \( w \) and vertical interval \( h \) in the VLP plane. In the VLP plane, we can use rectangles with the same height \( h \) and width \( w \) in order to cover the mineralized exploration works and ensure that the center of each rectangle is located in an exploration work. Thus, the total number of the mineralized rectangles is \( C_a \). The area \( A_i \) of each rectangle can be expressed by:

\[
A_i = wh. \tag{1}
\]

The mineralized area or orebody area \( A \) can be considered as the sum of the areas of all the mineralized rectangles, thus:

\[
A = C_a A_i = C_a wh. \tag{2}
\]

We can multiply the grade–thickness \( l \) of mineralized exploration works by the width \( w \) and height \( h \) in order to obtain the metal tonnage \( M_i \) in each rectangle. Similarly, we multiply the orebody thickness \( t_i \) of exploration works by the width \( w \) and height \( h \) in order to get the ore tonnage \( O_i \) in each rectangle. We sum the local metal tonnage \( M_i \) in all the rectangles in order to obtain the global metal tonnage \( M \) in the mineralized area and, similarly, to obtain the global ore tonnage \( O \) (Fig. 1c).

The global metal tonnage \( M \) and global ore tonnage \( O \) can be expressed, respectively as:

\[
O = \sum_{i=1}^{C_a} O_i = \sum_{i=1}^{C_a} wh t_i = wh \sum_{i=1}^{C_a} t_i = A \frac{C_a}{C_a} \sum_{i=1}^{C_a} t_i \tag{3}
\]

\[
M = \sum_{i=1}^{C_a} M_i = \sum_{i=1}^{C_a} wh l_i = wh \sum_{i=1}^{C_a} l_i = A \frac{C_a}{C_a} \sum_{i=1}^{C_a} l_i. \tag{4}
\]

2.3. Fractal algorithms

2.3.1. Number–size model

The variables, including grade–thickness and orebody thickness in exploration works, can be assumed to conform to the following number–size model proposed by Mandelbrot (1983):

\[
N(\geq r) = Cr^{-D} \tag{5}
\]

where \( r \) represents the orebody thickness \( t_i \) or grade–thickness \( l_i \) in exploration works. \( N(\geq r) \) stands for the cumulative number of exploration works in which the variable is not less than \( r \). \( D \) is called a fractal dimension. \( C \) is a capacity constant and is equal to \( N(\geq 1) \); it represents the number of objects, the sizes of which are not smaller than 1.

Eq. (5) can also be rewritten as:

\[
\ln N(\geq r) = -D \ln r + \ln C. \tag{6}
\]

In a diagram displaying the cumulative number plotted against the size in \( \ln \)-\( \ln \) coordinates, i.e., \( \ln N(\geq r) \) versus \( \ln r \) plot, Eq. (6) results in a straight line with slope \(-D\). Other common distributions (such as the normal (Gaussian), log-normal or negative exponential) yield distinctly curved graphs of \( \ln N(\geq r) \) versus \( \ln r \) plot.

If the plots of \( N(\geq r) \) and \( r \) in \( \ln \)-\( \ln \) coordinates can be fitted with one straight line, the distribution is called a simple fractal; in the case that the plots can be fitted with several straight-line segments, the model is called a bifractal, which means that the fractal models are different in each segment. In a bifractal model, the size range is divided into several segments, with threshold \( R_i \) (\( i = 1, 2, \ldots, n \)), dimension \( D_i \) and capacity constant \( C_i \) for the \( i \)th segment (Fig. 2).

A threshold \( R_i \) is a value of \( r \) that demarcates two adjoining straight-line segments. The \( R_i \) can denote \( T_i \) representing thresholds in the orebody thickness distribution or thresholds \( L_i \) representing thresholds in the grade–thickness distribution. The distribution of natural objects often shows bifractal characteristics. In a bifractal model, there is often a sharp change from one segment to the next, and the threshold can be easily selected; when the change is slow, we can determine the threshold by adjusting it to achieve the highest sum of the regression coefficients of the adjacent fitted lines.

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2.3. Fractal model for metal tonnage estimation

For simple fractal and bifractal models, the ore volume can be expressed, respectively, as:

\[ V = \sum_{i=1}^{n} \frac{\rho \cdot w \cdot C_i \cdot D_i}{1 - D_i} \left( T_{i+1} - T_i - D_i \right) \]
\[ \text{where } T_1 \text{ is } t_{\text{min}} \text{ and } T_n \text{ means } t_{\text{max}}, \text{ and } C_n \text{ is expressed by:} \]
\[ C_n = \prod_{i=2}^{n} T_i^{D_i - D_{i-1}}. \]

Ore tonnage in a bifractal model is then expressed by:

\[ O = \rho wh \sum_{i=1}^{n} C_i D_i \left( T_{i+1} - T_i - D_i \right). \]

We can further re-write Eq. (14) as:

\[ O = \rho wh \sum_{i=1}^{n} C_i D_i \left( T_{i+1} - T_i - D_i \right) \left[ \frac{D_i}{1 - D_i} \left( T_{i+1} - T_i - D_i \right) \right] + \sum_{i=2}^{n} \frac{\rho wh C_i D_i \prod_{j=2}^{n} T_j^{D_j - D_{j-1}}}{1 - D_i} \left( T_{i+1} - T_i - D_i \right). \]

Considering Eq. (8), Eq. (15) becomes:

\[ O = \rho wh \sum_{i=1}^{n} C_i D_i \left( T_{i+1} - T_i - D_i \right) \left[ \frac{D_i}{1 - D_i} \left( T_{i+1} - T_i - D_i \right) \right] + \sum_{i=2}^{n} \frac{\rho wh C_i D_i \prod_{j=2}^{n} T_j^{D_j - D_{j-1}}}{1 - D_i} \left( T_{i+1} - T_i - D_i \right). \]

Eq. (16) still denotes that global ore tonnage is proportional to orebody area.

2.3.3. Fractal model for metal tonnage estimation

If the distribution of grade-thickness in exploration works on the VLP plane follows the number-size model, we can deduce the fractal models for the metal tonnage estimation. The fractal models of metal tonnage for simple fractal and bifractal models can be expressed, respectively, as:

\[ M = \rho wh \sum_{i=1}^{n} C_i D_i \left( T_{i+1} - T_i - D_i \right) \left[ \frac{D_i}{1 - D_i} \left( T_{i+1} - T_i - D_i \right) \right] + \sum_{i=2}^{n} \frac{\rho wh C_i D_i \prod_{j=2}^{n} T_j^{D_j - D_{j-1}}}{1 - D_i} \left( T_{i+1} - T_i - D_i \right). \]

where \( L_i \) is the minimum grade-thickness \( l_{\text{min}} \) and \( L_n \) is the maximum grade-thickness \( l_{\text{max}} \) of the dataset.

Eqs. (16) and (18) represent that the reserve is associated with the orebody area but is not related to the number of exploration works.

2.3.4. Grade estimation

After calculating metal tonnage and ore tonnage of deposit, we can estimate ore grade \( G \) in the whole deposit following the Eq. (19):

\[ G = \frac{M}{O}. \]

2.4. Limitations of the FMRE

2.4.1. Exploration grid density

Exploration works are often unevenly distributed in space. In order to reach a reserve with higher controls and classifications, exploration grids in shallow parts are often made denser than those in deep parts, where grids are designed for obtaining the reserve with greater quantity.

However, both sparser and denser grids can represent the overall distribution of mineralization variables in an orebody area, although denser grids can reflect the actual orebody geometry and fractal distribution of mineralization variables to a greater extent and thus achieve a more accurate estimation result. Therefore, the number-size model obtained via a sparser grid should be analogous to that based on a denser grid; therefore, we can estimate the global reserve based on exploration works distributed in the overall orebody area, with the precondition that the geological mineralization characteristics in the area are similar.

2.4.2. Ore density

In most deposits, ore density proportionally increases with mineral grade. If the ore industrial type is the same, ore density varies in a small range. Therefore, the FMRE uses the average density of ores to calculate the reserve, as is the same with the GBM. Therefore, in the application of the FMRE, it is necessary to estimate separately the reserve of ores of different industrial types.

2.4.3. Variable limit

As metal prices increase, or as technology enables exploitation of an orebody with lower grades or smaller thicknesses, the \( r_{\text{min}} \) declines (Annels, 1991). Because the variables are often clustered in a small range, the small variance of \( r_{\text{min}} \) can induce a critical change to orebody area and affect the calculation result significantly.

Many fractal models of geological objects show that the fractal dimension often increases from the first segment to the ith segment in a bifractal model (Roberts et al., 1998; Monccke et al., 2001; Deng et al., 2009; Wan et al., 2010). Therefore, \( D_n \) is often large enough such that the item \( (1 - D_i) \) is smaller than zero. For \( 1 - D_n < 0 \), \( r_{\text{max}}^{D_n} < 1 \) and \( r_{\text{max}} > 1 \), and the small variance of \( r_{\text{max}} > 1 \) produces little effect on \( r_{\text{max}}^{D_n} \) value, further on the estimation result.

2.5. Steps in the FMRE

Three steps are required in the FMRE. The first step is delimiting orebody area. Two approaches are proposed to delimit orebody area. In the first one, we delimit the outlines of orebodies by directly connecting the marginal mineralized exploration works. The second one is the same as that used in the GBM, in which orebody outlines are extended outwards for a certain distance from the marginal exploration works with greater mineralization intensity, and the inside waste boundaries are also obtained in a similar way; then, orebody area is obtained as the difference between the area confined by the orebody outlines and the waste area. Selecting between the two approaches is decided by mineralization continuity as judged by the geologists.

In the next step, we calculate the fractal parameters of mineralization variables in the overall orebody area. Finally, according to the FMRE, we estimate the global reserve within the whole exploration area. A numerical example involving calculations using Eqs. (16) and (18) is presented in the following case study.

3. Case study

3.1. Regional geology and deposit geology

3.1.1. Regional geology

The Dahong phonite ore deposit is located in the Jiaodong Peninsula, China (Fig. 3). The Jiaodong Peninsula is famous for its large gold reserve.
Gold mineralization consists of K-feldspar alteration, silicification and contains gold mineralization. The alteration related to fault and divides the deposit into southern and northern parts (Fig. 4). The Dayingezhuang fault in the central of the deposit crosscuts the Zhaoping 20° to 51° controls the alteration and mineralization of the deposit. The Linglong granite in the footwall. The Zhaoping fault with variant dips from 0 to 80° has a tendency to be parallel with the Linglong granite, which is considered to be related to the mineralization of the deposit. The Precambrian sequence is composed of basement rocks of the Late Archean Jiaodong Group, which consists of mafic to felsic volcanic and sedimentary rocks metamorphosed to amphibolite to granulite facies. Plutonic rocks, which intruded into the Precambrian basement in the northwestern part of the Jiaodong Peninsula, have been traditionally divided into two suites, the Linglong and the Guojialing. The Linglong suite consists of medium-grained metaluminous to slightly peraluminous biotite granite, and the Guojialing suite is composed of porphyritic hornblende-biotite granodiorite. The ages of emplacement of these granitoid suites are 160–156 Ma and 130–126 Ma (SHRIMP U–Pb zircon data; Wang et al., 1998; Qiu et al., 2002), respectively.

The gold province consists of three ore-bearing fault zones, i.e., the Sanshandao zone, the Jiaojia zone, and the Zhaoping zone, from west to east in the northwestern part (Fig. 3). More than half of the gold reserves are concentrated in the Zhaoping gold belt. Gold deposits in the northwestern part of the Jiaodong Peninsula are divided into two types: the quartz vein style and the disseminated-veinlet style (Fig. 3). Gold deposits consisting of disseminated veinlets dominate the province (Deng et al., 2005, 2006, 2008b; Yang et al., 2006, 2007). Direct Rb–Sr dating of pyrite from the major Linglong vein-style gold deposit in the Jiaodong district shows that gold mineralization occurred at about 123–122 Ma (Yang and Zhou, 2001). Zhang et al. (2003) dated sericite at 121.3 ± 0.2 Ma by 40Ar–39Ar methods in the disseminated-veinlet style Cang-shang gold deposit, which is located in the same fault zone as the Sanshandao deposit.

3.1.2. Deposit geology

The Dayingezhuang disseminated-veinlet ore deposit is in the middle segment of the Zhaoping fault. Wallrocks in the Dayingezhuang deposit comprise the Jiaodong Group in the hanging wall of the Zhaoping fault and Linglong granite in the footwall. The Zhaoping fault with variant dips from 20° to 51° controls the alteration and mineralization of the deposit. The Dayingezhuang fault in the central of the deposit crosses the Zhaoping fault and divides the deposit into southern and northern parts (Fig. 4).

In the footwall of the Zhaoping fault, a cataclastic altered zone develops and contains gold mineralization. The alteration related to gold mineralization consists of K-feldspar alteration, silicification, sericitization, chloritization, phyllic alteration, sericite-quartz alteration and carbonatization. The degree of fracturing and alteration gradually weakens outwards from the Zhaoping fracture plane. There are more than 100 orebodies of different sizes in the Dayingezhuang ore region. Among these orebodies, orebody I–1 in the southern part and orebody II–1 in the northern part of the Dayingezhuang gold deposit are the biggest and take up most of the gold reserves.

3.2. Deposit explorations and raw data

In the Dayingezhuang gold deposit, more than 430 exploration works (mostly drifts, some drills) at different exploration lines and at different depths have been completed up to the end of 2007. Geological and mineralization characteristics are generally consistent at different depths in orebody I–1 and orebody II–1.

Since the orebody dip of the deposit is around 45°, both the VLP and HVP are applicable. In this paper, we adopt the VLP. In the VLP plane, the exploration works fall into the depths range from −13 m to −830 m. The distribution of exploration works is uneven; it generally becomes sparser from top to bottom (Fig. 5).

Via the GBM, the reserves in orebody I–1 and orebody II–1 are calculated with a cutoff of 1 g/t and a minimal grade–thickness of 0.8 m g/t. The orebody area, ore tonnage and metal tonnage of orebody I–1 are 485,434 m², 9.0 Mt and 28.72 t, respectively; those in orebody II–1 are 572,457 m², 22.02 Mt and 78.63 t. About 107 t metal and 31.03 Mt ore in the two orebodies have been explored, within an area of 1,057,891 m² (Table 1). In this paper, the global ore reserve in the deposit is the sum of the reserves in both orebody I–1 and orebody II–1. It is considered that the outlier in gold assay values in an exploration work may represent a high-grade zone with limited spatial extent. The zone is spatially discontinuous and impossible to predict in production scheduling (Parker, 1991). Therefore, the outliers are substituted by the average ore grade of the exploration work in the reserve calculations.

The data of exploration works come from the Dayingezhuang ore deposit. Total 417 mineralized exploration works in both orebody I–1 and orebody II–1 are utilized in this paper. As in the GBM, the grade outliers are substituted by the average ore grade during estimation grades in each exploration work. The histograms of the natural logarithms of grade–thickness and thickness for these exploration works are illustrated in Fig. 5a and c, and the ‘quantile–quantile’ plots (Q–Q plot) in Fig. 6a and d. The Q–Q plots show that the variables do not follow a log-normal distribution, especially in the two tails, indicating skewed distribution. It will be shown that the skewed distributions of either grade–thickness or thickness can be described by a bifractal model. It is further shown in
Figs. 7 and 8 that changes in thickness and in grade–thickness along vertical and horizontal directions are both abrupt and irregular.

3.3. FMRE application

3.3.1. Consistence test of the fractal model

Before calculating the reserves via the FMRE, we test the consistency of the fractal distribution of the mineralization variables obtained with various grid densities. For the test, we can divide the area with uneven exploration grids into several sub-areas with even exploration grids and calculate the fractal models in the different sub-areas; alternatively, we can calculate and compare the fractal models of the different areas at various depths, i.e., including several grid areas with different densities, as shown in Fig. 9.

The grade–thickness plots and thickness plots in ln–ln coordinates both show bifractal characteristics comprising four or three segments (Fig. 9). With increasing depths, the plots in the fractal models in both Fig. 9a and b become denser, further verifying that the grade–thickness and orebody thickness can be considered continuous variables when the number of exploration works is sufficiently high. Overlaying the grade–thickness or thickness plots at different depths together reveals that patterns of plot distributions are similar (Figs. 9–11).

Fractal parameters for grade–thickness and orebody thicknesses in exploration works above −200 m, −300 m and −830 m, are listed in Tables 2 and 3. From the first segment to the fourth (Table 2) or the third (Table 3), the fractal dimension increases. In the four fractal models for the orebody grade–thickness, the fractal dimension of the first segment varies from 0.08 to 0.10; that of the second segment, from 0.35 to 0.52; that of the third segment, from 0.97 to 1.45; and that of the fourth segment, from 8.22 to 10.22. In the fractal models for orebody thickness, the fractal dimension of the first segment varies from 0.33 to 0.36; that of the second segment, from 0.89 to 0.98; that of the third segment, from 1.75 to 8.37.

The fractal dimensions of the corresponding segments for the four grade–thickness fractal models vary in a small range, and the fractal dimensions of the orebody thickness distribution show more variance.
than those of the grade–thickness distribution. This suggests that the thickness and grade–thickness distribution is approximately consistent in the deposit, from the well-explored region to the less-explored region.

3.3.2. Reserve estimation

The orebody area is calculated via the two methods. The area obtained by directly connecting the mineralized exploration works is 880,481 m², which is smaller than that obtained from the GBM, in which the orebody outlines are extended outwards for a certain distance from the marginal mineralize exploration works.

Based on the orebody area estimated via the GBM and the fractal parameters, the global ore tonnage and global metal tonnage are calculated and listed in Tables 2 and 3. The fractal model based on the group of overall exploration works, i.e., those above −830 m, is suitable for the global estimation of the deposit, since this group can reflect the true mineralization distribution in the global orebody area more comprehensively than the other three groups, in which exploration works are only partly distributed.

As shown in Table 2, the estimated metal tonnage obtained by the fractal model for depth down to −830 m is 107.66 t and those according to the fractal models for depths down to −200 m, −300 m

<table>
<thead>
<tr>
<th>Orebody number</th>
<th>Orebody area (m²)</th>
<th>Density (t/m³)</th>
<th>Grade (g/t)</th>
<th>Ore tonnage (Mt)</th>
<th>Metal tonnage (t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No.I-1</td>
<td>485,434</td>
<td>2.84</td>
<td>3.19</td>
<td>9.00</td>
<td>28.72</td>
</tr>
<tr>
<td>No.II-1</td>
<td>572,457</td>
<td>2.76</td>
<td>3.57</td>
<td>22.02</td>
<td>78.63</td>
</tr>
<tr>
<td>Sum</td>
<td>1,057,891</td>
<td>2.78</td>
<td>3.46</td>
<td>31.03</td>
<td>107.35</td>
</tr>
</tbody>
</table>

Fig. 6. Histograms and Q–Q plots of the raw data utilized in the case study. (a) Histogram for grade–thickness; (b) Q–Q plot for grade–thickness; (c) histogram for thickness; (d) Q–Q plot for thickness.

Fig. 7. Grade–thickness and orebody horizontal thickness curves at different levels within exploration lines 80 and 80.5 in Dayingezhuang ore deposit. (a) Grade–thickness curve; (b) orebody horizontal thickness curve.
and −400 m are 82.35 t, 110.78 t and 111.10 t with the relative errors with respect to the metal tonnage based on the fractal model for depth down to −830 m are 23.51%, 2.90% and 3.19%, respectively. The ore tonnage obtained by the fractal models for depth down to −830 m is 30.06 Mt. The ore tonnage obtained by the fractal models for depths down to −200 m, −300 m, −400 m are 25.46 Mt, 30.04 Mt, and 30.16 Mt (Table 3), with relative errors relative to the ore tonnage obtained by the fractal models for depth down to −830 m are 15.30%, 0.06% and 0.33%, respectively. The estimated average grade of the deposit is 3.58 g/t.

It is clear that the estimates based on the fractal models for depths down to −300 m, −400 m and −830 m are somewhat larger than the estimates derived from the fractal models for depth down to −200 m. This can be explained by the fact that data in exploration works down to depth of −200 m have less statistical meaning and cannot represent all the mineralization features of the orebodies in their entirety. Exploration works at depth down to −200 m are in the marginal part of the orebody area, and the mineralization intensity is weaker, thus inducing a higher fractal dimension in the second and third segments and further resulting in reduced reserve estimates. The estimated ore tonnage is more unstable than the metal tonnage, probably because orebody thickness is more variable than grade–thickness in the disseminated–veinlet deposit.

The fractal models of the grade–thickness in orebody I-1 and in orebody II-1 are illustrated separately in Figs. 12a and 13a, and the fractal models of the orebody thickness are shown in Figs. 12b and 13b respectively. The reserve in orebody I-1 and orebody II-1 is estimated and listed in Tables 2 and 3. The grades calculated via the FMRE of the orebody I-1 and II-1 are 2.97 g/t and 3.51 g/t, respectively.

The sum of the metal tonnage in orebodies I-1 and II-1 is 107.73 t, which is roughly equal to the global metal tonnage 107.66 t obtained via the FMRE, with relative error 0.06% (Table 2). The sum of the global ore tonnage in orebodies I-1 and II-1 is 32.17 Mt, which is roughly equal to the ore tonnage 30.06 Mt obtained via the FMRE, with a relative error of 7.01% (Table 3).

4. Discussion

4.1. Estimation result

The ore tonnage and metal tonnage in the Dayingezhuang ore deposit are calculated via the FMRE proposed in this paper, since the
Fig. 10. Fractal models of grade–thickness at different depths in the Dayingezhuang deposit, China. The different colors represent the different size segments. (a), (b), (c) and (d) represent the fractal models of data in exploration works for depths down to $-200$ m, $-300$ m, $-400$ m and $-830$ m, respectively.

Fig. 11. Fractal models of orebody horizontal thickness at different depths in the Dayingezhuang deposit, China. The different colors represent the different size segments. (a), (b), (c) and (d) represent the fractal models of data in exploration works for depths down to $-200$ m, $-300$ m, $-400$ m and $-830$ m, respectively.
distributions of local orebody thickness and grade–thickness can be described by bifractal models. It is documented that the roll-off effect is a common feature in fractal analyses and is usually attributed to neglecting the samples at smaller scales or with smaller sizes (Walsh et al., 1991; Blenkinsop and Sanderson, 1999). However, in some cases with more systematic samples, the roll-off effect can be avoided. For instance, the element distribution along the drifts with varied mineralization intensity in the Dayingezhuang deposit invariably follows the bifractal model. Because, in the exploration grid, all the exploration works are carefully analyzed, we can consider that the fractal analysis can diminish the roll-off effect and reflect the true distribution, as is confirmed by the consistency of the fractal models in various areas of the Dayingezhuang deposit.

The estimation result with the fractal parameters of the overall exploration works is accepted for the global reserve in the Dayingezhuang deposit. The calculated results via the FMRE and the geometric and geostatistical methods, which are based on linear mathematics, assume that mineralization variables vary evenly throughout an orebody. However, the local outliers or anomalies are invariably developed, and produce great effects on the estimation process and result of the traditional methods. The traditional methods need to replace the outliers via different ways even if they are the true values, or to apply complex data processing to diminish the influence of the outliers (Anik, 1990). The FMRE utilizes the nonlinear fractal model to fit the skewed distributions of mineralization variables instead of changing the outliers or utilizing complex data processing. Hence, the FMRE can deal with the natural variability and skewness more effectively, which is the essential difference between the FMRE and the traditional methods. In the case study of the FMRE, the distributions of orebody thickness and grade–thickness are skewed, and they are directly fitted by a bifractal model.

Preliminary data processing is the same in the FMRE and GBM. In the two methods, we need to deal with the outliers that should not be involved in reserve estimation; then project exploration works onto a vertical or horizontal plane and delimit orebody area according to orebody thickness and grade in each exploration work. Yet, the later steps of data processing in the FMRE are much easier than the corresponding later steps in the GBM.

The obvious disadvantage of the FMRE is that it can only estimate global reserve in a deposit; in contrast, the geometric and geostatistical methods can estimate both local reserve and global reserve.

### 4.2. Comparison between FMRE and traditional methods

The geometric and geostatistical methods, which are based on linear mathematics, assume that mineralization variables vary evenly throughout an orebody. However, the local outliers or anomalies are invariably developed, and produce great effects on the estimation process and result of the traditional methods. The traditional methods need to replace the outliers via different ways even if they are the true values, or to apply complex data processing to diminish the influence of the outliers (Anik, 1990). The FMRE utilizes the nonlinear fractal model to fit the skewed distributions of mineralization variables instead of changing the outliers or utilizing complex data processing. Hence, the FMRE can deal with the natural variability and skewness more effectively, which is the essential difference between the FMRE and the traditional methods. In the case study of the FMRE, the distributions of orebody thickness and grade–thickness are skewed, and they are directly fitted by a bifractal model.

### 4.3. FMRE application scope

Just like the GBM, the FMRE application is based on the VLP of exploration works. Since the exploration process in deposits of different genetic types, e.g., porphyry deposit, quartz vein deposit, SEDEX deposit, VMS deposit, etc., are almost similar, a system of exploration works is required. The FMRE proposed in this paper deals with the distribution of orebody thickness and grade–thickness in exploration works and have no relationship with the genetic type of

### Table 2

Metal tonnage calculated by the FMRE with the orebody area obtained via the GBM in the Dayingezhuang deposit, China.

<table>
<thead>
<tr>
<th>Orebody area (m²)</th>
<th>Density (t/m³)</th>
<th>Fractal parameters</th>
<th>Total metal tonnage (t)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>D₁ D₂ D₃ D₄ L₄ L₆</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a</td>
<td>1,057,891</td>
<td>2.78</td>
<td>0.33 0.34 0.69 1.75 67</td>
</tr>
<tr>
<td>b</td>
<td>1,057,891</td>
<td>2.78</td>
<td>0.33 0.34 0.69 1.75 67</td>
</tr>
<tr>
<td>c</td>
<td>1,057,891</td>
<td>2.78</td>
<td>0.33 0.34 0.69 1.75 67</td>
</tr>
<tr>
<td>d</td>
<td>1,057,891</td>
<td>2.78</td>
<td>0.33 0.34 0.69 1.75 67</td>
</tr>
<tr>
<td>No.I orebody</td>
<td>572,457</td>
<td>2.76</td>
<td>0.33 0.34 0.69 1.75 67</td>
</tr>
<tr>
<td>No.II orebody</td>
<td>485,434</td>
<td>2.84</td>
<td>0.33 0.34 0.69 1.75 67</td>
</tr>
</tbody>
</table>

a, b, c and d represent the fractal models of orebody thickness comprising the exploration works upon −200 m, −300 m, −400 m and −830 m, respectively.

### Table 3

Ore tonnage calculated by the FMRE with the orebody area obtained via the GBM in the Dayingezhuang deposit, China.

<table>
<thead>
<tr>
<th>Orebody area (m²)</th>
<th>Density (t/m³)</th>
<th>Fractal parameters</th>
<th>Total ore tonnage (Mt)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>D₁ D₂ D₃ D₄ L₄ L₆</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
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<tr>
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<td>b</td>
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<td>c</td>
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<td>2.78</td>
<td>0.33 0.34 0.69 1.75 67</td>
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<td>No.II orebody</td>
<td>485,434</td>
<td>2.84</td>
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</tr>
</tbody>
</table>

a, b, c and d represent the fractal models of orebody thickness comprising the exploration works upon −200 m, −300 m, −400 m and −830 m, respectively.
deposits. Therefore, the FMRE can be widely applied to various genetic types of deposits if orebody mineralization variables in exploration works conform to a fractal model.

Compared to conventional estimation methods, the FMRE is more suitable for deposits explored with irregular drillhole densities or with highly skewed distributions of mineralization variables or lacking of proper geologic modeling.

5. Conclusions

The traditional geometric and geostatistic methods based on linear mathematics are difficult for dealing with the skewed distribution of mineralization variables, which is the common characteristic of mineralization, and thus require complex data processing. Fractal models belonging to nonlinear mathematics are effective tools for describing the natural variability and skewed distribution of geological objects. Based on the fractal number–size model, we established a new reserve estimation tool, called fractal models for reserve estimation (abbreviated as FMRE), in this paper. In the FMRE, we first calculate orebody area according to...
mineralized exploration works projected onto a VLP plane, and then estimate the fractal parameters of the distribution of mineralization variables. After obtaining orebody area and the fractal parameters of orebody thickness, global ore tonnage can be derived by the FMRE; similarly, global metal tonnage can be estimated based on the orebody area and the fractal parameters for grade-thickness. In the FMRE, mineralization variables are considered to be continuous, which means that distributions of data obtained from a sparse grid are analogous to those derived from a dense grid and can represent the overall distribution of the variables; this indicates that the orebody area is filled with exploration works with infinitely small intervals, and thus the estimated results denote the actual ore tonnage and metal which can be mined out without consideration of the ore loss rate in the deposit. The Dayingezhuang gold deposit in China was used as a case study; this revealed that the estimated reserves via the FMRE and those derived from traditional geometric block methods are very similar, with relative error 3.11% in ore tonnage and 0.29% in metal tonnage. The case study further demonstrates that the fractal distribution obtained via a sparse grid is similar to that derived from a dense grid.

For proper application of the FMRE, we should be aware that: (1) denser exploration grids can give more accurate fractal model and orebody area, and thus better estimation results; (2) that reserves of ores of totally different industrial types or densities should be estimated separately; and (3) that the selection of the lower limit of the variables based on cutoff grade and minimum mining thickness can influence orebody area and estimated reserves to a greater extent.

The FMRE can be applied not only to gold deposits but also to other types of mineral deposits. In addition, the FMRE are much easier than traditional geometric block methods to implement because the FMRE allow.

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