Tonnage-cutoff model and average grade-cutoff model for a single ore deposit

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ABSTRACT

Ore reserve estimation and resource prediction are important aspects of deposit exploration and mining. The cutoff has great influence on the reserve and resource calculation. Despite this, the study on the mathematical relationships of the tonnage, average grade and cutoff is still scant. According to the fractal distribution of element concentrations, which are obtained from analysis of channel samples of constant length along exploration or mining works, the tonnage-cutoff and average grade-cutoff models are constructed, which are applicable in both exploration and operating mine environments. Via these models, the relationships between ore tonnage, metal tonnage and cutoff are obtained in terms of the fractal model of element concentrations. Moreover, the thickness of the mineralized zone in each exploration or mining work follows a fractal model, and assuming that the thickness is a continuous variable, a model for calculating the mass of the mineralized zone in a single deposit is established. Given the mineralized zone mass, the tonnage-cutoff and average grade-cutoff models can be utilized to calculate ore reserve and average grade, especially when the cutoff changes. A fault-controlled disseminated-and-veinlet gold deposit in Jiaodong gold province, China, is selected as a case study. The estimates of the ore reserves obtained from these models are found to be consistent with the results obtained by the traditional geometric block method.

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1. Introduction

The tonnage, average grade, and cutoff are the basic parameters in mineral resource assessment and mining operations. Research on the inherent relationships between the tonnage, average grade above cutoff and the cutoff has been carried out for decades on the scale of an ore camp, district or ore province. During both exploration and mining environments within an individual deposit, it is also an essential task to continually study relationships between the average grade, tonnage, and cutoff (Taylor, 1985; Minnitt, 2004). Such research not only benefits understanding of the essence of metallogenesis in geological terms, but serves as a basis for further exploration or mining. For instance, although the cutoff is often stable during an on-going mining operation, it may also be adjusted due to any major long-term increase or decrease of metal price, or other change in market conditions, which induce modifications of the deposit tonnage and average grade above cutoff. Despite the volume of previous works, studies on the tonnage-cutoff model and average grade-cutoff model for a single ore deposit are still scant. In this paper, it is proposed that models can be established based on the element concentrations obtained from the mineralized zones exposed by exploration or mining works.

1.1. Traditional methods for ore tonnage and average grade estimation

Traditional methods for ore tonnage and average grade estimation in a single ore deposit can be roughly classified as either geometric or geostatistical (Henley, 2000). In exploration, both of these methods utilize the orebody thickness and the grade over a given thickness (e.g., in drillholes or drifts) located in a grid to estimate the reserve. In traditional methods, as a cutoff is given, the orebody thickness and average grade above cutoff of each exploration work are calculated, and the corresponding ore tonnage and average grade of the orebody and ore deposit are estimated. This calculation process is repeated each time the cutoff changes to obtain the relationships between ore tonnage, average grade and cutoff. Theoretical principles and practical applications of the traditional methods have served mining for decades, and have become much easier with the aid of modern programming (David, 1974). The explicit formulae describing the inherent relationships between ore tonnage, average grade and cutoff are, however, rarely studied.

1.2. Models of grade, tonnage and cutoff

The distributions and relationships of the tonnage, grade and cutoff have been studied for decades to predict resources at a regional scale, generally involving a larger number of mineral deposits (Musgrove, 1965; David, 1974; Harris, 1984; Gerst, 2008). USGS researchers studied the grade frequency distribution model and tonnage frequency distribution model for various types of ore deposits, and used the models to...
estimate resource potential at the regional scale (Singer, 1993, 1995; Singer et al., 2000, 2005). Ore tonnage of the deposits in a given region is proven to follow a fractal distribution (Barton and La Pointe, 1995; Turcotte, 2002). Lasky (1950) demonstrated that the grade and the logarithms of cumulative ore tonnage above a given cutoff conform to a linear relationship. In addition, a lognormal relationship between tonnage and cutoff was also introduced by Musgrove (1965). Matheron (1959) further pointed out that Lasky’s model is derived from the lognormal distribution of the ore grades. DeYoung (1981) proved, however, that Lasky’s equation has mathematical limitations; it was discovered that a plot of cumulative metal tonnage versus log cumulative ore tonnage from Lasky’s model has a maximum, which implies that the grade becomes ‘negative’ beyond the maximum value. This is an undesirable mathematical property for any practical tonnage-grade model. Therefore, Lasky’s model is not widely applicable. In contrast, Cargill et al. (1980) obtained a fractal (power–law) relationship for tonnage and average grade by plotting the cumulative tonnage against the grade. Turcotte (1997) further developed a tonnage-grade model, which results in a fractal relationship between tonnage and grade, by modeling the geological ore-forming process.

It is suggested that the fractal model is an effective tool to describe the relationship between tonnage and grade at the lognormal model at a regional scale. The mathematical relationships between tonnage, grade and cutoff have, however, rarely been investigated within a single deposit.

1.3. Fractal distributions of geological objects

Geological objects and variables normally show irregular, heterogeneous, and skewed characteristics. These complex characteristics can be described by fractal models. The number-size model is one of the most widely applied fractal models (Mandelbrot, 1983; Cheng et al., 1994; Agterberg, 1995; Wang et al., 2008, 2010b; Deng et al., 2010). Orebody thickness and grade-thickness in the exploration workings of a single deposit were described by the number-size model (Wang et al., 2010a). Additionally, the element concentrations in precious and base metal deposits also conform to the number-size model. For example, Sanderson et al. (1994) showed that Au grades in the La Codosa quartz vein deposit, Spain, follow the number-size model; Monecke et al. (2001) noted that base metal concentrations in drillholes from the Hellyer massive sulfide deposit, Tasmania, Australia, also obey the model. Monecke et al. (2005) showed that the frequency distributions of Zn, Pb, Cu, and Ag in the Waterloo massive sulfide deposit, Australia, conform to the fractal model. Roberts (2005), Wang et al. (2007) and Deng et al. (2009) discovered that the Au concentrations in some disseminated-and-veinlet deposits in Australia and China can be described by the fractal model.

1.4. Research objective

It is proposed that the fractal model is suitable for describing the relationship between tonnage and grade at a regional scale, and the distribution of element concentrations in a single ore deposit. Based on the number-size fractal model, this paper aims to work out the formulae of the relationships between ore tonnage and cutoff, and between average grade above cutoff and the cutoff. The Shangzhuang disseminated-and-veinlet Au deposit, Jiaodong province, China, (Deng et al., 2008) is selected as a case study.

2. Raw data and geological models

In the exploration or operating mine environments of a deposit, a series of exploration or mining works are performed. The following modeling is based on such workings in an exploration environment; by analogy, the same process is nevertheless also applicable in an operating mine environment.
In the GBM, one of the most popular methods for reserve estimation, the first step is to define the orebody range with area $A$ in the VLP or HLP. The orebody area can then be manually separated into various blocks by exploration workings in vertices of the VLP or HLP, and the inside average grade, horizontal or vertical thickness of orebody; the local reserves in each block are calculated according to the exploration intersections. Finally, the global reserve in a deposit is obtained by summing up the local reserve in each block (Annels, 1991).

2.2. Model for calculation of mineralized zone mass

The raw data utilized in the models established in this paper are the same as that used in the GBM. The following modeling in this paper is performed in an HLP; modeling in a VLP can be considered analogous.

The vertical mineralization thickness $u_i$ of each exploration working is obtained, and they are projected onto HLP. We assume that degree of exploration is evenly distributed according to a grid (Fig. 1b). In the HLP, the intervals of the exploration workings in the two perpendicular directions are $w$ and $l$, respectively, and $w$ and $l$ are assumed to be constant in the modeling. We use rectangles with length $l$ and width $w$ to cover each exploration working to ensure the center of each rectangle is located in an exploration working. The total number of exploration workings is denoted as $N_c$. The orebody area $A$ in the HLP can be considered as the sum of the areas of all the rectangles (Fig. 1b), thus:

$$A = N_c w l$$

(1)

2.3. Geological model for tonnage-cutoff model

For another view, we can use cuboids with the same length $h$, width $w$ and height $e$ to cover the channel samples in 3-D space and ensure that the center of each cuboid is located in a channel sample (Fig. 1c). Based on the cuboids, the mineralized zone in 3D can be identified (Fig. 1c). Given a particular cutoff, the outlines of the orebody are further determined in the mineralized zone.

Assuming that the mineralized area has a nearly uniform density $\rho$, the mineralized zone mass can be considered as:

$$O_\alpha = N_c \rho w h e$$

(2)

and the ore tonnage is expressed as:

$$O = N_c \rho w h e$$

(3)

where $N_\alpha$ is the total number of channel samples and $N_c$ is the number of channel samples with element concentrations no less than the cutoff $C_c$.

3. Number-size model

The number-size model can be expressed by the equation:

$$N(\geq r) = C r^{-D}; C > 0, D > 0$$

(4)

where $r$ represents the element concentrations of the channel samples $g_i$ or the local thickness of mineralized zone $u$, $N$ denotes the number of objects that are equal to or greater than the scale $r$, $C$ is a constant representing the size of the dataset and equal to $N(\geq 1)$, and $D$ is the fractal dimension.

Eq. (4) can be rewritten as follows:

$$\ln N(\geq r) = -D \ln r + \ln C$$

(5)

In a diagram displaying the cumulative number plotted against the size in ln-ln coordinates, i.e., a plot of $\ln N(\geq r)$ versus $\ln r$, Eq. (5) determines a straight line obtained by a least-square regression with slope $-D$.

If the plots of $N(\geq r)$ and $r$ in ln-ln coordinates can be fitted with one straight line, the distribution is called a simple fractal model. In the case where the plots are fitted with several straight line segments, the model is called a bifractal, which implies that the fractal dimensions are different in each segment. In a bifractal model, the range of sizes is divided into several segments, with threshold $R_i$ ($i = 1, 2, ..., n$), and dimension $D_i$ and capacity constant $C_i$ for the $i$th segment (Fig. 2). In the bifractal models for the element concentration $g_i$ and mineralized zone thickness $u$, the thresholds are separately denoted as $G_i$ and $U_i$. Both $G_i$ and $U_i$ are uniformly expressed by the notation $R_i$. The $C_i$ is expressed by the following equation:

$$C_i = C_1 \prod_{j=2}^{i} R_j^{D_j-D_1-1}$$

(6)

In a bifractal model, there is often a sharp change from one segment to the next, and the threshold can be easily selected; when the change is moderated, we can determine the threshold by adjusting it to maximize the sum of the regression coefficients of two adjacent fitted line segments.

In a fractal model, when the minimum value $r_{min}$ of the dataset is equal to 1, the capacity constant $C$ is equal to the total number $C_0$ of the objects. When $r_{min}$ is not equal to 1, we can use the fractal dimension $D$ and the constant $C$ to calculate the total object number $C_0$ in a simple fractal model by (7):

$$C_0 = C r_{min}^{-D}$$

(7)

and that in a bifractal model by Eq. (8):

$$C_0 = C r_{min}^{-D}$$

(8)

$C_0$ can represent the total number of exploration workings $N_c$, the number of all channel samples $N_\alpha$, and the number of the channel samples with concentration greater than the cutoff $C_c$, respectively.
where \( r_{\text{min}} \) is equal to the lowest mineralized zone thickness, the lowest concentration and the cutoff.

4. Calculation of the mineralized zone mass

According to fractal models for reserve estimation established by Wang et al. (2010a), by assuming that the mineralized zone thickness follows a simple fractal distribution and it is a continuous variable when \( w \) and \( l \) are sufficiently small, the mineralized zone mass \( O \) is thus expressed as:

\[
O_{a} = \int_{u_{\text{min}}}^{u_{\text{max}}} \rho \text{wh} \frac{dN(\geq w)}{du} (1-D) \left[ u_{\text{max}}^{1-D} - u_{\text{min}}^{1-D} \right]
\]

(9)

According to Eq. (1), we rewrite Eq. (9) as:

\[
O_{a} = \rho \text{wh}N_{u} D_{u_{\text{min}}}^{D} \left[ u_{\text{max}}^{1-D} - u_{\text{min}}^{1-D} \right] = \rho A D_{u_{\text{min}}}^{D} \left[ u_{\text{max}}^{1-D} - u_{\text{min}}^{1-D} \right]
\]

(10)

where \( u_{\text{min}} \) is the minimum mineralized zone thickness, \( u_{\text{max}} \) is the maximum, \( \rho \) is the average density of the mineralized rocks and \( A \) is the mineralized area in the HLP obtained by GBM.

For bifractals, the mass of the mineralized zone is deduced as:

\[
O_{a} = \rho A D_{u_{\text{min}}}^{D} \left[ D_{1} \left( u_{1}^{1-D_{1}} - u_{1}^{1-D_{1}} \right) + \sum_{i=2}^{n} D_{i} \prod_{j=i}^{n} \left( u_{j}^{1-D_{j}} - u_{1}^{1-D_{1}} \right) \right]
\]

(11)

where \( U_{1} \) is \( u_{\text{min}} \) and \( U_{n} \) is \( u_{\text{max}} \).

5. Tonnage-cutoff model

If the element concentrations of the channel samples follow a simple number-size model, then by combining Eqs. (2) and (4), the mass of the mineralized zone can be expressed as:

\[
O_{a} = \rho \text{wh}N_{g} = \rho \text{wh}C_{g_{a}}^{D}
\]

(12)

where \( C_{g_{a}} \) is the lowest non-zero concentration of the channel samples. Similarly, based on Eq. (3), the ore tonnage can be expressed as:

\[
O = \rho \text{wh}N_{g} = \rho \text{wh}C_{g_{D}}^{D}
\]

(13)

Based on Eqs. (12) and (13), we have:

\[
O = O_{a} \left( \frac{C_{g_{a}}}{C_{g_{D}}} \right)^{-D}
\]

(14)

In the bifractal model, the mass of the mineralized zone is

\[
O_{a} = \rho \text{wh}C_{g_{a}}^{D}
\]

(15)

Assuming that \( C_{D} \) is in the i-th segment of the bifractal model, the ore tonnage can be expressed as:

\[
O = \rho \text{wh}C_{g_{D}}^{D}
\]

(16)

and combining Eqs. (6), (15) and (16), the ore tonnage can be evaluated from

\[
O = M_{g_{a}}^{D_{D}} C_{g_{D}}^{D}
\]

(17)

6. Average grade-cutoff model

According to the simple number-size model in Eq. (4), assuming that the concentration \( g \) is a continuous variable, the average grade above cutoff \( G_{m} \) of the ore can be expressed as:

\[
G_{m} = \frac{\int_{g_{d}}^{g_{D}} \rho g dN(\geq g) (1-D) \left[ (1-D_{1}) C_{g_{a}}^{D} - (1-D) C_{g_{max}}^{D} \right]}{C_{g_{max}}^{D} - C_{g_{min}}^{D}} (D > 0, D \neq 1)
\]

(18)

where \( G_{D} \) is the cutoff and \( G_{max} \) is the maximal element concentration.

In a bifractal model with \( n \) segments, assuming that \( C_{g_{D}} \) is in the i-th segment, \( G_{m} \) can be written as:

\[
G_{m} = \frac{C_{g_{D}}^{D} (1-D_{i+1}) C_{g_{D}}^{D} - C_{g_{D}}^{D} (1-D_{i}) C_{g_{D}}^{D}}{C_{g_{max}}^{D} - C_{g_{min}}^{D}}
\]

(19)

where \( G_{D} \) is \( G_{max} \).

7. Applications of the tonnage-cutoff model and average grade-cutoff model

These models not only display the relationships between the mineralized zone mass, ore reserve, average grade and cutoff, but also provide a new method for calculating the mineralized zone mass, ore reserve and average grade. These models are applicable in both exploration and operating mine environments.

The established models can reveal the relationships between mineralized zone mass and ore reserve, between average grade above cutoff and the cutoff, between ore tonnage and cutoff, and therefore between ore tonnage and average grade above cutoff. Therefore, the influence of changes in the cutoff on the ore reserve and average grade can be readily studied. These proposed models are applicable in the different deposit styles, if only the distribution of element concentrations and mineralized zone thickness in the deposit conform to the bifractal model.

In the case that the intervals of the exploration works are constant in the deposit, we can directly apply Eqs. (13) and (16) to estimate the ore tonnage beyond a given cutoff. If the exploration works are irregular or uneven, we can calculate the mass of mineralized zone using Eq. (10) or (11), and further estimate the ore tonnage based on Eq. (14) or (17). Whereas, when the number of workings in the exploration environment is small, the fractal model of element concentrations of the samples is assumed to reflect the concentration distribution in the whole exploration area. Then the volume and mass of the mineralized zone can be roughly estimated in terms of the geometric information obtained from exploration data and the geologists’ observations and inference, the resource in the deposit is predicted finally.

Three steps are required in these models. First we need to determine the element concentrations in the exploration intersections distributed evenly or randomly in the mineralized zone and calculate the number-size model of the concentrations. Second, based on the number-size model, we obtain the relationships between average grade and cutoff, between ore tonnage and cutoff, and between ore tonnage and the average grade above cutoff by assuming the mineralized zone mass is a constant. Finally, we estimate the mass of the mineralized zone and determine the ore reserve in the ore deposit. It should be noted that the calculation of mineralized zone mass can be also put before step two.

8. Case study

The Shangzhuang deposit, a fault-controlled disseminated-and-veinlet deposit located in the Jiaodong Au province, China, was selected for the case study.
8.1. Geological setting

The Jiaodong gold province lies on the Jiaodong Peninsula of eastern China. Three NE-trending first-order ore-bearing fault zones occur from west to east across the province. These are the Sanshandao zone, Jiaojia zone, and Zhaoping zone. Disseminated-and-veinlet Au deposits, characterized by high tonnages, wide alteration zones, and multiple alteration types, are dominant in the province (Deng et al., 2006, 2008; Yang et al., 2006, 2007). Using \(^{40}\text{Ar}–^{39}\text{Ar}\) methods, Zhang et al. (2003) dated sericite at 121.3 ± 0.2 Ma in the Cangshang disseminated-and-veinlet Au deposit.

The Shangzhuang deposit is located in the central segment of the Wangershan fault zone, a secondary fault of the Jiaojia fault (Fig. 3). Nearly 30 tonnes of gold has been identified above the −600 m level in the last 30 years. The main rocks in the deposit are Linglong-type granite (medium-grained metaluminous to slightly peraluminous biotite granite) and Guojialing-type granite (porphyritic hornblende-biotite granodiorite) (Fig. 3), with emplacement ages of 160–156 Ma and 130–126 Ma, respectively (Lü and Kong, 1993; Wang et al., 1998; Qiu et al., 2002; Deng et al., 2006).

In the deposit, alteration and mineralization are confined to the Wangershan fault. The alteration zone, which is 1800 m in length and 30 to 50 m in width, strikes N25° to 45°E and dips NW at 30° to 40° (Fig. 4). The alteration styles related to Au mineralization consist of K-feldspar alteration, silification, sericitization, chloritization, phyllic alteration, sericite-quartz alteration and carbonatization. Mineralization is closely spatially related to sericite-quartz alteration. Element concentrations in the mineralized zone vary greatly, resulting in irregularly shaped orebodies; ores occur as stockworks, veinlets or disseminations. The main minerals within the ores include pyrite, pyrrhotite, sericite and K-feldspar, with lesser magnetite, chlorite, siderite, ankerite, epidote, chalcopyrite, galena, sphalerite and, locally, minor arsenopyrite and marcasite. Gold occurs mainly as gold and electrum in pyrite, with minor free gold and silver in the altered host rocks.

8.2. Deposit exploration and raw data

An exploration area in the southern part of the Shangzhuang deposit was selected for the case study. Ten drillholes were completed in the area (Fig. 5). Channel samples with length of nearly 1 m were collected along the drillhole and the Au concentrations analyzed. The orebody thickness and grade in each drillhole are calculated assuming that the cutoff is selected as 1.5 g/t. After the exploration intersections are projected to a HLP, the orebody area is determined and its area is estimated to be 21,359.29 m². Via the GBM, the ore tonnage is estimated to be 1.63×10⁵ t, and the mean Au grade to be 3.28 g/t.

A total of 316 Au assays from the 10 exploration intersections are utilized in the case study. A histogram of the natural logarithms of Au concentrations, and the respective ‘quantile–quantile’ (Q–Q) plot are...
shown in Fig. 6. The Q–Q plots show that the concentrations do not follow a lognormal distribution. The fractal models will be applied to describe the concentration distribution.

8.3. Mineralized zone mass estimation

The lowest non-zero value in the Au concentrations is 0.1 g/t, which is taken as \( G_{\text{a}} \). According to the lowest concentration, the thickness of the mineralized zone in each drillhole is calculated as shown in Fig. 5. The fractal model for the mineralized zone thickness in the drillholes is illustrated in Fig. 7. According to the fractal model and Eq. (11), the mineralized zone mass is estimated to be \( 1.69 \times 10^6 \) t.

8.4. Relationships between tonnage, average grade and cutoff

The Au concentration data in the drillhole samples are fitted with three straight lines in the fractal model, as shown in Fig. 8. By Eq. (17),

\[
\ln N(\geq u) = a + \frac{b}{u}
\]

the ore tonnage–cutoff relationship is obtained and illustrated in Fig. 9a. Based on Eq. (19), the average grade and cutoff curve is as shown in Fig. 9b. Therefore, the metal tonnage–cutoff curve and tonnage–average grade curve are achieved, as shown in Fig. 9c and d, respectively.

When the cutoff was taken to be 1.5 g/t, the ore tonnage was calculated as \( 1.56 \times 10^5 \) t, which is consistent with the result derived from the GBM with relative error 4.5%. Moreover, the average grade was estimated to be 3.24 g/t, with relative error 1.2% compared to that obtained by GBM.

9. Discussions and conclusions

In this paper, via the fractal model, basic equations are established for ore tonnage estimation, also for the relationships between tonnage and cutoff, between average grade and cutoff, and thus between ore tonnage and average grade. These equations are applicable in both exploration and operating mine environments.

Assuming that the mineralized zone thickness in each exploration or mining work conforms to the number-size model, the model for the calculation of mineralized zone mass is established. Via another viewpoint, assuming that the element concentrations in the channel samples in the mineralized zones exposed by the exploration or mining works conform to the number-size model, two models are set

![Fig. 6. Histogram and Q-Q plots of the Au concentrations in the drillholes in the southern part of the Shangzhuang deposit. (a) Histogram; (b) Q-Q plot.](image)

![Fig. 7. Fractal model of the mineralized zone thickness in the ten selected drillholes in the southern part of the Shangzhuang deposit. The plots in the same segment in the bifractal model are represented by one uniform symbol (triangle, rectangle or rhomb).](image)

![Fig. 8. Fractal model of gold concentrations in the channel samples in the ten selected drillholes in the southern part of the Shangzhuang deposit. The plots in the same segment in the bifractal model are represented by one uniform symbol (triangle, rectangle or rhomb).](image)
up to calculate the mineralized zone mass and ore tonnage, respectively. Based on these two models, it is deduced that the ore tonnage and the division of cutoff and the lowest element concentration value show a fractal relationship with the same fractal dimension as the number-size model of element concentrations. It is thus inferred that the larger the fractal dimension of the element concentration distribution, the greater the decrease rate of ore tonnage as the cutoff increases. Based on the same number-size model of element concentrations, the average grade-cutoff model is deduced. Ultimately, the curve between the ore tonnage and average grade can be obtained.

Given that the element concentrations are evenly or randomly distributed in the mineralized zone, the relationship between ore tonnage and cutoff and that between average grade and cutoff can be obtained. In addition, when the number of exploration or mining works is large and the mineralized zone can be easily delimited, the models proposed in this paper can be applied to estimate ore tonnage. Furthermore, in grassroots or preliminary exploration, while ore intersections may be rare, the potential resource can be predicted by roughly estimating the mineralized zone mass.

A fault-controlled disseminated-and-veinlet gold deposit in Jiaodong gold province, China, is selected for a case study. The ore tonnage, metal tonnage, average grade and cutoff are estimated and the relationships between them are obtained. The estimates of ore reserves obtained from the models proposed in this paper are consistent with those obtained by the traditional geometric block method. The comparisons indicate that the new mathematical modeling is a reliable approach for studying tonnage–cutoff relationships.

Compared to classical and standard methods, the new approach provides explicit formulae. From this point, these new models can describe the inherent relationships between tonnage, average grade above cutoff and the cutoff, and should be applied much easier with the aid of programming. Despite this, the new methods fail to calculate the local reserve, which must still be derived using traditional estimation methods.

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