A generalized procedure to identify three-dimensional rock blocks around complex excavations

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SUMMARY

This paper presents a generalized procedure for the identification of rock blocks formed by finite-sized fractures around complex excavations. It was assumed that the study domain could be partitioned into a finite number of subdomains, where each either was, or could be, approximated by a convex polyhedron, and the fractures were finite in size and disc shaped and were defined using the location of the disc center, orientation, radius, cohesion coefficient, and friction angle. These may be either deterministic fractures obtained from a field survey or random fractures generated by stochastic modeling. In addition, the rock mass could be heterogeneous; i.e. the rock matrix and individual fractures could have different parameters in different parts. The procedure included: (1) partitioning of the model domain into convex subdomains; (2) removing noncontributive fractures. A fracture was deemed contributive when it played a part in block formation; i.e. it formed at least one surface with some of the blocks; (3) decomposing the subdomains into element blocks with fractures; (4) restoring the infinite fractures to finite discs; and (5) assembling the modeling domain. Our procedure facilitates robust computational programming, and is flexible in dealing with the problem of a complex study domain and with rock heterogeneity. A computer code was developed based on the algorithm developed in this study. The algorithm and computer program were verified using an analytical method, and were used to solve the problem of block prediction around the underground powerhouse of the Three Gorges Project. Copyright © 2008 John Wiley & Sons, Ltd.
1. INTRODUCTION

A procedure to identify three-dimensional (3D) rock blocks formed by fractures and excavation surfaces is a desired technique in rock engineering. Most often it is needed as a practical tool for many rock structures, as the potential instability of rock blocks is a major concern, as discussed in Goodman and Shi [1], Hoek et al. [2], and Harrison and Hudson [3]. Such a tool would be very helpful for engineers who are often faced with the problem of how to determine the number, location, dimensions, and stability of rock blocks around a high rock slope or a large underground cavern before excavations can take place.

The development of such techniques is also stimulated by the wide application of the discrete element method (DEM) and discontinuous deformation analysis (DDA) techniques [4–8]. In the modeling of the dynamic movements of block systems, both DEM and DDA are needed to describe the geometry of the blocks explicitly.

Some authors have discussed the issues in block identification. In initial works, the fractures were assumed to be infinitely large and hence, the blocks were limited to convex shapes [9, 10]. Lin et al. [11] and Ikegawa and Hudson [12] presented their block identification approaches for finite-extended fractures, based on topological concepts. In these approaches, the intersections of the fractures are calculated first to define the sets of vertices, edges, and faces. Then, the sets were regularized to discard any isolated and dangling vertices, edges, and faces. Finally, the blocks were identified through boundary chain operations of the closed surfaces and the Euler–Poincare formula for a polyhedron. Similar methods have also been discussed in detail by Jing and Stephansson [13], Jing [14], Ohnishi and Yu [15], and Lu [16].

In the above studies of the identification of blocks formed by finite fractures, the fractures can be either contributive or noncontributive to the formation of blocks. This makes them distinguished from the Block Generation Language model [10, 17], where the fractures always form fully connected network and all the fractures are completely contributive.

In this study, the fractures can cross each other and end in intact rock. This assumption has been commonly adopted by the studies of block identification formed by finite fractures [11–16], and is also suggested by the mapped fractures presented by Kulatilake et al. [18], Doughty et al. [19], and Starzec and Andersson [20].

This paper presents a new procedure, whose important advantage is its simplicity. It decomposes a complex block into a finite number of convex element blocks during the identification process. The adoption of the concept of an element block makes it possible to represent a complex block using an assemblage of simple convex blocks, and thus, to transform the difficult calculations involved in identifying the blocks around complex excavations to simple calculations of a convex geometry.

The complexity of the modeling domains and the heterogeneity of a material have been the major reasons for the development of various numerical methods. These two issues have been extensively discussed using the finite element method (FEM) and the finite difference method (FDM) techniques. However, they have rarely been mentioned in work involving block theory and in methods of rock block identification. These two problems are fully taken into consideration in the method discussed in this paper.

The major objective of this study was to develop a practical tool for static rock block analysis and to support the design of rock structures in blocky rocks, determining the location, dimensions,
and stability of individual blocks when the geometry of an excavation and the fractures are well characterized. The identifying procedure used the following assumptions:

- The modeling domain can be partitioned into a finite number of subdomains, and that each of these subdomains either was (or could be approximated) a convex polyhedron shape.
- The fractures are finite in extent and are disc shaped, and that they are well defined mathematically in terms of their location, orientation, extent, cohesion, and friction angle.
- The rock mass may be either heterogeneous or homogeneous. The heterogeneity of the rock mass includes the variation in the properties that characterize the rock matrix (e.g. density) and those properties that characterize the fractures. For example, different parts of an individual fracture can have different friction angles.
- An individual block is a rock body completely enclosed by fractures and boundary faces (excavated surfaces, natural outcrops, or artificial faces to separate the regions of enclosed from the infinite rock mass).

Since it is almost impossible to access rock masses in three dimensions, the real 3D shapes of fractures are rarely investigated directly. In fact, fractures may have arbitrary complex shapes. This often forms an obstacle when using 3D fracture models. Consequently, many authors (e.g. Kulatilake et al. [18], Zhang and Einstein [21], and Priest [22]) assume that the fractures are planar and have the same extent in all directions: i.e. the fractures are disc shaped. This assumption is a numerical simplification, likely valid for massive igneous rocks. It may be invalid in some cases. For example, in layered sedimentary rocks, the short fractures that intersect the bedding planes usually terminate against the bedding planes. It is difficult to suggest that those short fractures are disc shaped.

In our procedure, a complex modeling domain is initially divided into convex subdomains, and each subdomain is further decomposed into convex blocks with infinite fractures. Then, the fractures are restored to finite dimensions, and the domain is reassembled by combining the subdomains into a full domain; i.e. the procedure comprises the following major steps:

- Subdividing the modeling domain into a finite number of convex subdomains.
- Removing the noncontributive fractures.
- Decomposing the subdomains into element blocks with the fractures, which are temporarily taken as infinite large at this stage.
- Restoring the infinite fractures to finite discs.
- Assembling the modeling domain.
- Classifying the complex blocks.

The element block is the basic unit used in the identification and the later computations of volume, weight, as well as the removability of the blocks. An element block has a simple convex shape and an invariant rock density. The mechanics coefficients are constant within the surface of an element blocks. Element blocks are formed when decomposing subdomains using fractures and are combined into complex blocks when restoring the fractures to finite discs. Complex blocks are assemblages of element blocks before reassembling the modeling domains. Only after the assembly of the modeling domain do complex blocks represent actual blocks enclosed by the fractures and excavations in rock masses.
2. SUBDIVIDING THE MODELING DOMAIN INTO SUBDOMAINS

For most actual rock structures, such as slopes or underground openings, subdividing the problem domain into a finite number of subdomains can be carried out either manually or automatically without any difficulty. An individual subdomain is a 3D solid with the shape of a convex polyhedron, and is defined by surface representation. That is, the subdomain solid is represented by its surface polygons, with each polygon defined by its boundary vertices that are arranged in either a clockwise or counterclockwise direction, and each vertex defined by its 3D coordinates.

Figure 1 shows a two-step slope. The domain of the slope can be decomposed into three subdomains. These subdomains are defined by 16 vertices and 16 polygons. Each of the subdomains is a polyhedron that is represented by six polygons, and each of these is defined by an oriented vertex list including four vertices.

The subdivision of a problem domain is very similar to the space discretization stage in FEM and the geometry and the associated data structure of a subdomain are also similar to those of an element in the FEM technique. The subdomains, surface polygons, and the vertices must be numbered uniquely, and no overlap among the subdomains or surface polygons is allowed. Table I shows the data structure for the subdomains of the slope shown in Figure 1.

The subdivision of a problem domain is easier than the discretization stage in FEM, because the dimensions of individual subdomains, as long as they are convex, have no effect on the correctness or the accuracy of the block identification. As will be revealed in the following sections, the computational requirement in block identification increases with the number of subdomains. Therefore, the number of subdomains should be kept as low as possible.

Curved excavation surfaces should be approximated using a given number of planar faces. Increasing the number of planar faces enhances the accuracy of the approximation, but also increases the number of subdomains, and hence, increases the computational requirement.

The rocks may have different densities in different subdomains. A large fracture, which may extend across different subdomains, can have different cohesion and/or friction angles in different subdomains. This realizes the heterogeneity of a rock mass in the problem domain. Consequently, the heterogeneity of a rock mass should be considered when subdividing problem domains, as a subdomain should not form across different heterogeneous regions.

According to their mechanical properties, the surfaces defining the subdomains can be divided into three types: (1) free faces, (2) fixed faces, and (3) common faces. The free faces are the actual excavated faces. The fixed faces are usually the faces that separate the domain of interest from

![Figure 1. Vertices and polygons used to define the two-step slope.](image-url)
Table I. Data structure for the sub-domains of the two-step rock slope.

<table>
<thead>
<tr>
<th>Surface no.</th>
<th>Sub-domain no.</th>
<th>Local no.</th>
<th>Global no.</th>
<th>Vertex list</th>
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the infinite rock mass. The free and fixed faces form the boundaries of the modeling domain. The common faces are shared by two neighboring subdomains on the opposite sides of the faces.

In the example shown in Figure 1, Faces 1, 2, 3, and 4, which are defined by vertex lists (1, 8, 16, and 9), (7, 15, 16, and 8), (6, 14, 15, and 7), and (5, 13, 14, and 6), respectively, are free. Faces 15 and 16, which are defined by vertex lists (4, 12, 15, and 7) and (2, 7, 15, and 10), respectively, are common-shared faces. The other faces of the 16 faces defining the subdomains are fixed.

The mechanical properties of the surfaces are used when classifying the blocks. Having at least one free face is a necessary condition for a removable block. If a block has one or more fixed faces, then it is not removable. If a block has a surface that is a part of a common-shared face, then the block is not an actual existing block, and is combined with other blocks to form a complex block, which is a real block enclosed by fractures and excavations.

3. REMOVING NONCONTRIBUTIVE FRACTURES

The fractures used in block analysis usually arise from excavated or natural outcrops mapping and sometimes they can even be from stochastic modeling. Whichever is the case, many fractures are often checked simultaneously to investigate the possibility of block occurrence. Among the many possible fractures, usually only a few contribute to the formation of blocks; i.e. they form facets of the blocks. Eliminating the noncontributive fractures before block identification lowers the computational requirements. This is particularly true when the fractures are generated from stochastic fracture modeling. As will be illustrated in Section 4 on block identification, the number of the element blocks increases with increasing number of fractures. This is the major reason why removing noncontributive fractures can reduce computational demand considerably.
In the identification method used in this study, it was not possible to distinguish the noncontributive fractures from the contributive fractures before diminishing the fractures and assembling the domain. However, some fractures intersected too few, or even no fractures, and so were obviously noncontributive, and they can be sought out and removed easily before block identification.

A contributive fracture forms the facets of the blocks. Hence, it must have at least three intersections with other fractures (and/or excavation surfaces), and these intersections must cross each other to form at least one closed loop in the plane of the fracture within the disc area that the fracture occupies.

The fractures shown in Figure 2 are common examples of noncontributive fractures. In Figure 2(a), the fracture has only two intersections. This type of noncontributive fracture can be found and removed by simply calculating and counting the intersections on it. In Figure 2(b), the fracture has four intersections, but these intersections do not form a closed loop. In Figure 2(c), the situation is more complicated. The fracture has more than three intersection lines and they form closed loops. However, the fracture is noncontributive. Among the five intersection lines on the fracture, the intersection lines $f_4$ and $f_5$, which are represented by dashed lines in Figure 2(c), are the intersection lines of the fracture with two fractures that have already been found to be noncontributive. After the removal of the two noncontributive fractures, the other intersection lines cannot form a closed loop.

The removal of noncontributive fractures is an iterative process. The elimination of previously found noncontributive fractures usually results in a decrease in the number of intersection lines with other fractures so that the intersection lines with them are less than three or they do not form a closed loop. The iterations are repeated until no new noncontributive fracture is found.

Determining whether or not the intersection lines in a fracture form closed loops includes the following steps: (1) calculation of the intersection lines of a fracture with other fractures, (2) if the number of the intersection lines is greater than two (otherwise, the fracture is noncontributive, and further calculations are not necessary), then transform the intersection lines from the global 3D
coordinate system to a 2D local system defined in the plane of the fracture, (3) calculation of the intersecting points among the 2D intersection lines, (4) iteratively find and eliminate the lines that have only a single intersecting point. If more than three lines remain after the iterative process, then the intersections with the fracture form at least one closed loop. As shown in Figure 2(b), there are four intersections with the fracture, and Intersection \( f_4 \) has only a single intersecting point. After the elimination of Intersection \( f_4 \), Intersection \( f_2 \) has only a single intersecting point. After the elimination of Intersections \( f_4 \) and \( f_2 \), the fracture has only two intersection lines on it, and it does not form a closed loop.

Note that the elimination procedure proposed here cannot ensure finding and eliminating all the noncontributive fractures, because having some closed loops of intersections is a necessary condition, and is not a sufficient condition for a fracture to form a facet of a block. However, because the purpose of the removal of the noncontributive fractures is to reduce the computational demand, then the remaining noncontributive fractures do not affect the correctness of the ensuing analysis. Therefore, in some cases, the step of removing the noncontributive fractures may be skipped safely. For example, when the fractures include only a small number of faults.

From a practical point of view, the fractures with a radius below a given criterion may be considered to be noncontributive and be eliminated. Usually, this criterion is determined artificially. For example, when identifying the blocks exposed on a slope, we removed all the fractures below 2 m in length. It was a cut slope about 30 m high and 100 m long. All the 252 visible fractures on the slope were mapped and observed. We found that almost all of the fractures below 2 m were isolated, or intersected with one or two fractures. Consequently, we removed all the 127 fractures below 2 m when performing block identification.

4. BLOCK IDENTIFICATION

In the process of block identification, each subdomain is initially decomposed into a finite number of convex blocks using fractures. At this stage, the fractures are temporarily visualized as infinite planes and the convex blocks are called element blocks. Then, the infinite fractures are diminished (restored) to finite-extended discs, and the subdomains are combined to reassemble the study domain.

4.1. Decomposing the subdomains into element blocks with fractures

Since a subdomain has a convex polyhedron shape, an infinite fracture will split the subdomain into two convex polyhedrons, as long as the fracture cuts the subdomain. To decompose a subdomain into element blocks, we can use the first fracture to split the subdomain into two smaller convex polyhedrons (if the fracture plane really cuts the subdomain, otherwise the subdomain remains full polyhedron). Then, we can use the second fracture to split the two polyhedrons resulting from the previous step. If the fracture cuts both polyhedrons, then they are split into four polyhedrons. In such a way, all the fractures are successively used to split the subdomain into element blocks.

The element block is the basic unit in the analysis procedures used in this study. In space, an individual element block is completely within one subdomain. This makes it easy to calculate accurately the weight, friction, and cohesion of the complex block to which the element block belongs for a heterogeneous rock mass. An element block is defined by a surface representation.
and is characterized using the following factors:

- The subdomain that the element block belongs to.
- The number of vertices and surfaces of the element block.
- A vertex list for each surface to record the vertices.
- The parameters of the plane equation for each of the surfaces of the element block (the parameters are preserved because they are used very frequently, e.g. when calculating the intersection between the block and a fracture plane).
- The 3D coordinates for each vertex of the block.
- The volume of the block and the area of each surface.
- An integer index for each surface to reflect the fracture (or excavation face) that forms the surface. This index is used when restoring fractures to the discs and when calculating the friction and cohesion of the complex block that the element block belongs to.
- An integer index for each surface, reflecting whether or not the surface is completely shared with another element block. A completely shared surface neither prevents the movement of, nor provides cohesion for, the complex block that the element block belongs to.

It would appear that the number of element blocks produced when decomposing the subdomains would increase exponentially, but this is not the case. Suppose an element block has \( n \) vertices, where \( v = \{v_i, i = 1, n\} \) is the vertex set of the element block, then in the 3D coordinate system, \( v_i = (x_i, y_i, z_i) \), and the plane equation of a fracture is given by \( ax + by + cz + d = 0 \), if \( ax_i + by_i + cz_i + d \geq 0 \) is always true. The entire block locates above the fracture plane, and the fracture does not split the element block. In contrast, if \( ax_i + by_i + cz_i + d \leq 0 \) is always true, then the entire block is located below the plane, and the fracture does not split the element block either.

Unnecessary splitting computations can also be avoided by using a bounding box test. Suppose a fracture’s radius is \( R \), then the location of its disc center is given by \( (X_o, Y_o, Z_o) \), and an element block’s bounding cube box is defined by \( (X_{\text{min}}, Y_{\text{min}}, Z_{\text{min}}) \) and \( (X_{\text{max}}, Y_{\text{max}}, Z_{\text{max}}) \). If one of the following three conditions is satisfied, then the fracture is distant from the element block, and so does not split the block:

\[
X_o + R \leq X_{\text{min}} \quad \text{or} \quad X_o - R \geq X_{\text{max}} \tag{1}
\]

\[
Y_o + R \leq Y_{\text{min}} \quad \text{or} \quad Y_o - R \geq Y_{\text{max}} \tag{2}
\]

\[
Z_o + R \leq Z_{\text{min}} \quad \text{or} \quad Z_o - R \geq Z_{\text{max}} \tag{3}
\]

where \( X_{\text{min}}, Y_{\text{min}}, \) and \( Z_{\text{min}} \) are the minimum coordinates of the vertices of the element block on the \( x-, y-, \) and \( z- \) axes, respectively, and \( X_{\text{max}}, Y_{\text{max}}, \) and \( Z_{\text{max}} \) are the maximum coordinates of the vertices of the element block on \( x-, y-, \) and \( z- \) axes, respectively.

More rigorous criteria to avoid unnecessary splitting computations may be implemented, but the computational savings are often offset by the computation of the criteria themselves.

4.2. Restoring infinite fractures to discs

In the previous step, the fractures were treated as being infinite, and they would all diminish to finite-extended discs whose geometry was defined by the location of their centers, orientation, and radius. This results in the combination of the element blocks into complex blocks.

For two element blocks, \( A \) and \( B \), if they share no common fractures, then there would be no possibility of combining them when diminishing the fractures. They would only combine when a
single fracture provides facets for both of them, that is,

\[ f(A_a) = f(A_b) = F \]  

(4)

where \( F \) is the fracture to be diminished, \( A_a \) and \( A_b \) are the facets of Blocks A and B, respectively, and \( f() \) is the function used to retrieve the fractures or excavations of the surfaces of the element blocks.

Blocks A and B must also be on opposite sides of the fracture plane. Suppose that the vertices of A are \( v_a = \{v_i, i = 1, n\} \), where \( v_i = (x_i, y_i, z_i) \), and the vertices of B are \( v_b = \{v_j, j = 1, m\} \), where \( v_j = (x_j, y_j, z_j) \). The plane equation of the common fracture, \( F \), is \( ax + by + cz + d = 0 \), and the following condition must be always true:

\[ (ax_i + by_i + cz_i + d)(ax_j + by_j + cz_j + d) \leq 0 \]  

(5)

Moreover, Facets \( A_a \) and \( A_b \) must overlap with each other either partly or completely, and the fracture disc does not circle the overlapped part completely. That is,

\[ A_{ab} \cup A_d \neq A_d \quad \text{and} \quad A_{ab} \neq \emptyset \]  

(6)

where \( A_{ab} = A_a \cap A_b \), \( A_d \) is the disc area of the finite fracture formed by diminishing the infinite fracture, \( F \), and \( \emptyset \) the empty set.

Figure 3 shows a typical situation for element block combination when diminishing the fractures. In Figure 3, Block A has four vertices, \( a_1, a_2, a_3, \) and \( a_4 \), and Block B has four vertices, \( b_1, b_2, b_3, \) and \( b_4 \). Surface \( a_2a_3a_4 \) (Facet \( A_a \)) of Block A and surface \( b_2b_3b_4 \) of Block B (Facet \( A_b \)) are on the same fracture plane. For Blocks A and B,

- In Figure 3(a), \( A_{ab} = A_a \cap A_b \neq \emptyset \), but \( A_{ab} \cup A_d = A_d \).
- In Figure 3(b), \( A_{ab} = A_a \cap A_b = \emptyset \), and \( A_{ab} \cup A_d = A_d \).
- In Figure 3(c), \( A_{ab} = A_a \cap A_b = \emptyset \), and \( A_{ab} \cup A_d = A_d \).
- In Figure 3(d), \( A_{ab} = A_a \cap A_b \neq \emptyset \), and \( A_{ab} \cup A_d \neq A_d \).
- In Figure 3(e), \( A_{ab} = A_a \cap A_b \neq \emptyset \), and \( A_{ab} \cup A_d \neq A_d \).

Consequently, in Figure 3(a), (b), and (c), the two element blocks do not combine, whereas in Figure 3(d) and (e) they do combine.

Because Blocks A and B are convex, \( A_a \) and \( A_b \) are convex polygons, and \( A_d \) is disc shaped. There is no difficulty in evaluating Equations (4), (5), and (6).

When restoring the fractures, each subdomain is handled independently, and thus, the combination only happens among the element blocks that belong to the same subdomain.

4.3. Assembling the domain

The element blocks combine into complex blocks when diminishing the fractures to finite discs. The complex blocks at this stage are not yet the blocks that actually exist in the rock mass, as they are groups of element blocks belonging to the same subdomains. Because each subdomain is treated independently when decomposing the subdomains into element blocks and diminishing fractures, the subdomains should be combined along the faces shared by neighboring subdomains to assemble the problem domain. This means that we eliminate the common faces of the subdomains, and lead to the combination of element blocks belonging to different subdomains.

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DOI: 10.1002/nag
The mathematical process for the combination of element blocks between different subdomains is fundamentally the same as that used for the combination of element blocks within the same subdomain caused by diminishing of the fractures.

For two element blocks, A and B, within two neighboring subdomains, these combine only when the face shared by the two subdomains provides facets for both of them. That is,

$$f(A_a) = f(A_b) = F$$  \hspace{1cm} (7)

where $F$ is the polygon face shared by the two subdomains, $A_a$ and $A_b$ are facets of Blocks A and B, respectively, and $f()$ is a function used to retrieve the fractures or the excavations of the surfaces of the element blocks.
Blocks A and B must also be on the opposite sides of a common shared face. This condition is satisfied naturally because A and B are in different subdomains. In addition, Facets $A_a$ and $A_b$ must overlap each other either partly or completely. That is,

$$A_a \cap A_b \neq \emptyset$$

Note that the combination of element blocks is progressive. The combination of one pair of element blocks can cause the combination of complex blocks that belong to different subdomains and can include multiple element blocks. Suppose a subdomain has a complex block $A = \{a_1, a_2, a_3, \ldots, a_i, \ldots, a_m\}$ and another subdomain has a complex block $B = \{b_1, b_2, b_3, \ldots, b_j, \ldots, b_n\}$. Then, if element blocks $a_i$ and $b_j$ combine, the complex blocks $A$ and $B$ combine to form a larger dimension complex block, $AB = \{a_1, a_2, a_3, \ldots, a_i, \ldots, a_m, b_1, b_2, b_3, \ldots, b_j, \ldots, b_n\}$. Moreover, the combination of element blocks at this stage may also cause a combination of the complex blocks in the same subdomain that have not combined during the diminishing of the fractures.

The complex blocks formed after assembling the domain are the final result of the block identification procedure, and they are the actual existing blocks in the rock mass isolated by fractures and excavations. A complex block may include one or more element blocks, and is defined by

- The number of the element blocks that comprise the block.
- Volume and weight of the block. They are simply the summarization of that of all the element blocks comprising the block.
- Two integers for each of the element blocks that comprise the complex block. One reflects the subdomain to which the element block belongs, and the other reflects the storage location of the element block within the list holding all the element blocks that belong to the same subdomain.

Additional parameters characterizing an individual complex block are: its removability type, sliding fractures, sliding force, friction, and cohesion. These are determined and assigned during the removability analysis of the block.

### 4.4. Classifying complex blocks

All the blocks formed by the boundary surfaces and fractures within the problem domain are identified using the above identifying procedure. Among these blocks, some are exposed at the free surfaces and others are surrounded or buried by other blocks. The exposed blocks are usually of more concern in the construction and design of rock projects. Some of these are removable in geometry, and others are nonmovable, because of the restrictions of the surrounding rocks. Consequently, these blocks are classified as shown in Figure 4.

![Figure 4. Classification of blocks.](image-url)
Goodman and Shi [23], Warburton [24], and Lin and Fairhurst [25] investigated a block's removability. When neglecting the rotation and deformation of a block, the moving direction of the entire block can be described using a unique tensor, \( s \). If we suppose that a block has \( N \) fixed surfaces formed by the fractures, then the tensor must meet the following condition:

\[
\mathbf{n}_i \cdot \mathbf{s} \geq 0 \quad (i = 1, \ldots, N)
\]

where \( \mathbf{n}_i \) is the unit normal of a fixed surface of the block and it points toward the inside of the block.

It can be seen that a removable block has at least one free surface formed by excavation and that it has no surface that is formed by fixed boundary face.

In addition to Condition (9), the vector \( \mathbf{s} \) must also coincide with the direction of the block's resultant driving force, that is,

\[
\mathbf{w} \cdot \mathbf{s} \geq 0
\]

If the driving force includes only the gravity of the block and its direction is \( \mathbf{w} = (0, 0, -1) \). If more than one directions meet Conditions (9) and (10), then the block moves along the direction in which \( \mathbf{w} \cdot \mathbf{s} \) takes a maximum value [25]. Physically, there are three possible forms of block movement: (1) falling along the direction of the resultant driving force, which is the direction of gravity if this is the only driving force, (2) sliding along a single fracture face, and (3) sliding along the intersection of two fractures; i.e. an edge of the block.

Determining the moving type of a block includes: (1) Investigating whether or not the block falls. If the block falls, then further computation can be saved. If all the normals of a block’s fixed surfaces point to the same half-space as the driving force; i.e. \( \mathbf{w} \cdot \mathbf{n}_i \geq 0 \) is always true, then the block will fall. (2) Determining whether or not a block can slide along a single fracture. Suppose that the plane equation of a fracture is given by \( ax + by + cz = d \), so that its falling line is given by \( \mathbf{f} = (ac, bc, -(a^2 + b^2)) \). If \( \mathbf{f} \cdot \mathbf{n}_i \geq 0 \) is always true, then the block slides along the falling line. (3) Suppose that a block edge (an intersection of two surfaces of the block) is equal to \( \mathbf{l} \), then if \( \mathbf{l} \cdot \mathbf{n}_i \geq 0 \) is always true and \( \mathbf{w} \cdot \mathbf{l} \geq 0 \), then the block slides along direction \( \mathbf{l} \).

If the block slides along a single fracture face then,

\[
F_s = W \sin \alpha
\]

\[
F_t = W \cos \alpha \tan \Phi
\]

\[
F_c = CA
\]

where \( F_s \), \( F_t \), and \( F_c \) are the sliding force, friction, and cohesion of the block, respectively. \( W \) is the gravity of the block, and \( \alpha \), \( \Phi \), \( C \), and \( A \) are the dip angle, friction angle, cohesion coefficient, and area of the sliding face, respectively.

If the block slides along an edge, then [23],

\[
F_s = |\mathbf{w} \cdot (\mathbf{n}_1 \times \mathbf{n}_2)| / |\mathbf{n}_1 \times \mathbf{n}_2|
\]

\[
F_t = |(\mathbf{w} \times \mathbf{n}_2) \cdot (\mathbf{n}_1 \times \mathbf{n}_2)| \tan \phi_1 + |(\mathbf{w} \times \mathbf{n}_1) \cdot (\mathbf{n}_1 \times \mathbf{n}_2)| \tan \phi_2 / |(\mathbf{n}_1 \times \mathbf{n}_2)|^2
\]

\[
F_c = C_1 A_1 + C_2 A_2
\]
where $F_s$, $F_f$, and $F_c$ are the sliding force, friction, and cohesion of the block, respectively, $w = (0, 0, -W)$, $n_1$, and $n_2$ are the normals, $\Phi_1$ and $\Phi_2$ are the friction angles, $C_1$ and $C_2$ are the friction coefficients, and $A_1$ and $A_2$ are the areas of the two sliding faces of the block, respectively.

The stability of a block may be defined by its factor of safety. For both of the above-mentioned sliding situations, the safety factors may be expressed as $f = (F_f + F_c)/F_s$.

In this study, a complex block was expressed as an assemblage of convex element blocks. This makes the method for movement direction searching, sliding face determining, as well as cohesion and friction computing different from those in the work of other authors.

Not all the surfaces of the element blocks belonging to the complex block form surfaces of the complex block. Some surfaces of the element blocks are shared with other neighboring element blocks. If a surface of an element block is shared completely with other element blocks, then the face is an interior face of the complex block, and is neither acting as a fixed face to prevent the movement of the block nor providing a sliding face or sliding edge for the block.

In the search for a sliding face or edge of a complex block, we can examine all the faces and edges of all the element blocks. To avoid redundant calculations, the faces and edges that have been investigated in the examination of an element block may be marked and can be skipped when examining other element blocks. For example, suppose Element block $A$ has a surface, $S$, that is formed by fracture $F$, and if $S$ is found not to be a sliding face of the complex block when checking $A$, then the surfaces formed with $F$ by the other element blocks may always be skipped in subsequent examinations.

According to Equations (11) and (14), the sliding force of the block is controlled by the sliding direction and is independent of the area of the sliding face. This means that no problem arises from expressing the sliding face as an assemblage of convex surfaces of several element blocks.

Suppose that $F$ is a fracture that forms a sliding face of a block and $A = \{a_1, a_2, \ldots, a_i, \ldots, a_m\}$ is the set of the element blocks that have a surface formed by fracture $F$. For the element blocks in $A$ the area of the parts of the surfaces that are formed by $F$ are marked and are not shared by any neighboring element blocks are $\{A_1, A_2, \ldots, A_i, \ldots, A_m\}$, and the cohesion coefficients of $F$ for the element blocks in $A$ are $\{C_1, C_2, \ldots, C_i, \ldots, C_m\}$. The cohesion provided by the sliding face that is formed by fracture $F$ is determined by

$$F_c = \sum A_i C_i \quad (i = 1, m)$$

The friction angle of the sliding face formed by fracture $F$ is determined by averaging the friction angles of the surfaces of the elements blocks weighted by the areas of the surfaces. Suppose that the friction angles for the element blocks in $A$ are $\{\phi_1, \phi_2, \ldots, \phi_i, \ldots, \phi_m\}$, then the average friction angle is

$$\phi = \sum A_i \phi_i / \sum A_i \quad (i = 1, m)$$

5. THEORETICAL VERIFICATION

A C++ computer program, known as GeneralBlock, was implemented based on the above algorithm. Readers interested in the software can download it from www.rockfractures.com. The algorithm and the computer program were verified using theoretical solutions. This designed various blocks formed by different intersection patterns of fractures, and the block identification process and removability analysis of the blocks were solved theoretically, so that the correctness
of the results from the computational program could be verified by comparing the data with the theoretical results. A detailed verification was carried out. Many examples were solved and the computational results compared with theoretical solutions. One of the examples is shown in Figure 5. The interesting point in the data shown in Figure 5 is that the excavations include both a slope and a tunnel and that the 3D block had been identified as being a hole.

In the example shown in Figure 5(a), the x-axis is orientated toward the south, the y-axis is orientated toward the east, and the z-axis is orientated upwards. The excavations include a tunnel and a three step slope. The lower two steps of the slope were 10 m high and the upper step was 5 m high. The benches of the slope were 5 m in width and the tunnel was cylindrical with a radius of 10 m, and its central axis ran parallel to the y-axis.

There were four fractures and their 3D coordinates (in m), dip direction, and dip angle were: \( f_1(25, 15, 6, 180^\circ/20^\circ) \), \( f_2(10, 15, 15, 180^\circ/90^\circ) \), \( f_3(20, 10, 15, 90^\circ/90^\circ) \), and \( f_4(20, 20, 15, 270^\circ/90^\circ) \), respectively. Their cohesion was neglected and the friction angle was 30°.

Analytical calculations demonstrate that the four fractures and the surfaces of the slope isolate a block, which is expressed by the hatched area shown in Figure 5(a). The tunnel segment between \( f_3 \) and \( f_4 \) is included in the block body as a 3D hole. When approximating the circular section of the tunnel with a regular dodecagon, the block had a volume of 2772.52 m\(^3\) (the hole in the tunnel was excluded), a weight of 70643.8 kN (density of rock = 2600 kg/m\(^3\)). \( f_1 \) is the sliding face of the block, the sliding force was 24161.4 kN, and the force of friction was 38326.5 kN. Thus, the factor of safety of the block was 1.59.

Figure 5(b) shows a 3D image of the block output in the block display dialogue window of the GeneralBlock program. The calculated block volume, weight, sliding fracture, sliding force, friction, and factor of safety are shown in the table located in the lower part of the window. As shown in the table, these have the same values as the analytical results. (Note that in the table, the unit used for the weight and force is a ton.) As expected, if the friction angle of fracture \( f_1 \) is 20°, then the factor of safety of the block is 1.0.

The computation error in block volume caused by approximating the circular tunnel section by a regular dodecagon was 1.3%. The approximation had no effect on the accuracy of the factor of safety of the block in this example.

6. APPLICATION

The above-mentioned procedure and our computer program were used to predict the blocks in the surrounding rock mass of the underground powerhouse of the Three Gorges Project. The powerhouse will be located on the right bank of the Yangtze River and will be constructed within the igneous rock mass under the Baiyanjian Hill. The powerhouse cavern is 333.6 m long and 86.24 m high. As shown in Figure 6, in the lower part of the powerhouse, the distance of the two vertical sidewalls is 31 m, and in the upper part, the span of the crown is 32.6 m.

Because of the large dimensions of the cavern, the rock blocks, especially the blocks formed by faults and excavation faces are cause for much concern. Consequently, the National Natural Science Foundation of China and the Changjiang Conservancy Commission founded this study in attempt to predict the behavior of the blocks.

To characterize the fractures, especially the large fractures in the rock mass around the cavern, more than 150 boreholes were drilled, and about 2600 m of exploration tunnels were excavated. Over half of the tunnels were concentrated at an elevation of 100 m above sea level (the altitude
Figure 5. Example to verify the algorithm and the computer program of block identification using theoretical solutions: (a) the fracture traces and the identified block (hatched area) using a theoretical method and (b) 3D graphics and the calculated results table of the identified block output using our computer program.
Figure 6. A cross-section of the underground powerhouse of the Three Gorges Project (the circled numbers denote the subdomains for block identification).

Figure 7. Locations of the 13 faults around the underground powerhouse.

of the bottom of the cavern is 19 m above sea level). Figure 7 shows an engineering geology map of the horizontal section at this elevation. As shown in Figure 7, 13 faults were found and investigated around the cavern.
Table II. Parameters of the faults around the powerhouse \((X_o, Y_o, Z_o=x, y, \text{and } z \text{ coordinates of the disc center, respectively.})\)

<table>
<thead>
<tr>
<th>Fault no.</th>
<th>(X_o) (m)</th>
<th>(Y_o) (m)</th>
<th>(Z_o) (m)</th>
<th>DipD (deg.)</th>
<th>Dip (deg.)</th>
<th>(R) (m)</th>
<th>(C) (MPa)</th>
<th>(\varphi) (deg.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f_1)</td>
<td>-28.516</td>
<td>15.625</td>
<td>81</td>
<td>255</td>
<td>85</td>
<td>50</td>
<td>0.175</td>
<td>31</td>
</tr>
<tr>
<td>(f_2)</td>
<td>-10.938</td>
<td>82.031</td>
<td>81</td>
<td>250</td>
<td>70</td>
<td>145</td>
<td>0.175</td>
<td>31</td>
</tr>
<tr>
<td>(f_3)</td>
<td>-17.968</td>
<td>120.703</td>
<td>81</td>
<td>250</td>
<td>70</td>
<td>65</td>
<td>0.175</td>
<td>31</td>
</tr>
<tr>
<td>(f_4)</td>
<td>-15.625</td>
<td>135.688</td>
<td>81</td>
<td>250</td>
<td>70</td>
<td>90</td>
<td>0.175</td>
<td>31</td>
</tr>
<tr>
<td>(f_5)</td>
<td>-9.766</td>
<td>183.594</td>
<td>81</td>
<td>250</td>
<td>70</td>
<td>115</td>
<td>0.175</td>
<td>31</td>
</tr>
<tr>
<td>(f_6)</td>
<td>0.001</td>
<td>185.547</td>
<td>81</td>
<td>250</td>
<td>72</td>
<td>110</td>
<td>0.175</td>
<td>31</td>
</tr>
<tr>
<td>(f_7)</td>
<td>-44.922</td>
<td>32.578</td>
<td>81</td>
<td>320</td>
<td>50</td>
<td>60</td>
<td>0.050</td>
<td>14</td>
</tr>
<tr>
<td>(f_8)</td>
<td>19.531</td>
<td>105.469</td>
<td>81</td>
<td>350</td>
<td>60</td>
<td>120</td>
<td>0.075</td>
<td>27</td>
</tr>
<tr>
<td>(f_9)</td>
<td>-7.031</td>
<td>156.25</td>
<td>81</td>
<td>345</td>
<td>65</td>
<td>95</td>
<td>0.085</td>
<td>27</td>
</tr>
<tr>
<td>(f_{10})</td>
<td>-39.063</td>
<td>145.39</td>
<td>81</td>
<td>354</td>
<td>84</td>
<td>85</td>
<td>0.085</td>
<td>27</td>
</tr>
<tr>
<td>(f_{11})</td>
<td>-23.438</td>
<td>221.875</td>
<td>81</td>
<td>350</td>
<td>58</td>
<td>40</td>
<td>0.085</td>
<td>27</td>
</tr>
<tr>
<td>(f_{12})</td>
<td>-33.203</td>
<td>226.172</td>
<td>81</td>
<td>0</td>
<td>78</td>
<td>45</td>
<td>0.085</td>
<td>27</td>
</tr>
<tr>
<td>(f_{13})</td>
<td>-42.578</td>
<td>237.109</td>
<td>81</td>
<td>10</td>
<td>78</td>
<td>50</td>
<td>0.085</td>
<td>27</td>
</tr>
</tbody>
</table>

The data from the exploration boreholes and tunnels demonstrated that the 13 faults were fairly smooth and planar. No data suggested that the faults had obvious variations in extent along different directions. Consequently, we assumed that the faults were planar disc shaped. In the analysis of block formation, only the 13 faults were investigated, other fractures of less scale were neglected.

Table II shows the geometric and mechanics parameters of the faults. In the block analysis, it was assumed that the fracture traces in Figure 7 are the diameters of the fracture discs and that the center of the fracture discs was located at the center of the traces.

As shown in Figure 6, in the analysis of the blocks, the study domain is subdivided into 20 subdomains that were defined by 101 polygons. Each of these was defined by four vertices. The 101 polygons were defined using 84 vertices, and the arched roof of the cavern was approximated using 16 line segments. The study domain was designed to be large enough such that the outside surfaces of the domain (expressed by the dashed lines in Figure 6) enclosed the fractures safely, and there was no intersection between the outside surfaces and the fractures. This prevented any unnecessary formation of blocks by the outside surfaces, which were artificially designed to separate the domain from the infinite rock mass.

After the elimination of noncontributive fractures, eight fractures remained: \(f_1, f_3, f_5, f_6, f_7, f_8, f_9,\) and \(f_{10}\), and these split the rock mass of the study domain into 299 element blocks. After diminishing the fractures into finite discs and reassembling the domain, the element blocks merged into four complex blocks. The largest block was the matrix rock mass for the other blocks in the domain, and this was not an actual block. Thus, it could be excluded from the real existing blocks. The block forming the matrix for the other blocks can be recognized by the characteristic surfaces they form with the boundaries that were artificially designed to separate the problem domain from the infinite rock mass.

Figure 8 shows the 3D graphics of the blocks and the cavern output using the GeneralBlock program. For the 3D coordinate system used, the \(xy\) plane coincides with the bottom of the cavern, and the \(x\)-axis is orientated in the direction of \(313.5\degree\), the \(y\)-axis is aligned parallel to the

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DOI: 10.1002/nag
Figure 8. 3D graphics of the blocks and the powerhouse cavern output using the GeneralBlock (in (b) the observer views the cavern downward vertically. In (c) the observer views the cavern horizontally along the positive x-axis direction).
longitudinal center axis of the cavern, and the z-axis is orientated vertically upwards. Figure 8(b) shows a parallel projection of the blocks and the cavern when the viewpoint is located at a distant point on the positive z-axis, while the observer views the cavern downwards vertically. Figure 8(c) shows a parallel projection of the 3D blocks and the cavern when the viewpoint is located at a distant point on the negative x-axis, and the observer views the cavern horizontally along the direction of positive x-axis.

According to our calculations using the GeneralBlock program, the 13 faults and the excavations form three blocks. All of them are removable blocks. The locations of them are shown in Figure 8(c).

Block 1 (B1 in Figure 8(c)), with a volume of 30523.1 m$^3$, is the largest of the blocks. It is formed by Faults $f_3$, $f_5$, $f_9$, and $f_{10}$, and it slides along the intersection between Faults $f_3$ and $f_{10}$. Its factor of safety is 1.361. The 3D coordinates of the block’s vertex that extends farthest into the rock mass, which is shown as Point $P$ in Figure 8(c), are $(-70.7, 127.7, 148.91)$.

Block 2 (B2 in Figure 8(c)) is 987.8 m$^3$ in volume. It is formed by Faults $f_5$, $f_6$, $f_9$, and $f_{10}$, and it slides along the intersection between Faults $f_5$ and $f_{10}$. Its factor of safety is 3.879.

Block 3 (B3 in Figure 8(c)) is 294.1 m$^3$ in volume, formed by Faults $f_1$, $f_7$, and $f_8$. It slides along the intersection between Faults $f_1$ and $f_8$. Its factor of safety is 4.696.

Note that the accuracy of the predicted results depends on the accuracy of the input data. It is difficult for current techniques to obtain accurate data on the location and the extent of fractures, and thus, the predicted results of blocks need to be verified and amended using data obtained from exploration and construction.

7. CONCLUSIONS

A procedure to identify 3D rock blocks has been presented. In this procedure, the fractures are finite in size and are disc shaped, with defined locations of the disc centers, orientation, radius, cohesion coefficient, and friction angle. They can be either deterministic fractures obtained from a field survey or random fractures generated from stochastic modeling. The rock mass can be heterogeneous, where the rock matrix and individual fractures can have different parameters in different parts.

The procedure is simple, and there is almost no complex concept, algorithm, or data structure in it. This makes it suitable for a robust computer program. A computer program based on this procedure was developed and verified using analytical solutions, and was applied to predict the rock blocks formed around the underground powerhouse of the Three Gorges Project.

The procedure is flexible, and the excavation can be an arbitrary complex shape and the rock mass in the problem domain can be heterogeneous.

The procedure is an actual data-based method and there are no artificial fractures or blocks. This is convenient for in situ engineers.

The fractures are assumed to be disc shaped, but there is no difficulty in extending the method to the fractures with other shapes, e.g. polygons.

For the method used in this work, the location, orientation, and dimensions of the fractures are necessary input parameters, as they fully determine the number, dimension, location, and removability of the blocks. This addresses the difficulties in field characterization of rock fractures.

ACKNOWLEDGEMENTS

This study was financially supported by the National Natural Science Foundation of China (Grant No. 40372134) and the Changjiang Conservancy Commission.

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DOI: 10.1002/nag

