An improved method for Q-factor estimates based on the frequency-weighted-exponential function

Chuanhui Li *, Xuewei Liu

Key Laboratory of Geo-detection (China University of Geosciences, Beijing), Ministry of Education, Beijing 100083, China

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The frequency-weighted-exponential (FWE) function was designed to fit asymmetric amplitude spectra by two parameters: symmetry index and bandwidth factor. It was applied to Q-factor estimates by fitting the amplitude spectra of source and attenuated wavelet. This method for Q-factor estimates was called the FWE method. The accuracy of the Q-factor estimates by the FWE method depends on the similarity between the modeled FWE functions and the amplitude spectra of source and attenuated wavelet. However, the amplitude spectra of source and attenuated wavelet are poorly fitted when the FWE function are modeled by measuring the symmetry index and bandwidth factor by their definitions. Hence we perform an improvement to the FWE method, where two FWE functions are employed to fit the amplitude spectra of source and attenuated wavelet by the Least Square Method to obtain the optimal symmetry index and bandwidth factor. The improved FWE method enhances the accuracy of the Q-factor estimates, and it also maintains the advantages of good applicability and tolerance to random noise of the original FWE method.

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1. Introduction

When seismic waves propagate through the earth, they undergo attenuation and dispersion as a result of the anelasticity and heterogeneity of the medium (Ricker, 1953; Futterman, 1962; Kneib and Shapiro, 1995). The Q-factor quantifies the absorption effect. It is employed to recover the frequency components of seismic waves with an inverse Q filter (Zhang and Ulrych, 2002; Wang, 2004). It is also a useful parameter for characterization of reservoirs and detection of hydrocarbons (Korneev et al., 2004; Li et al., 2006; Pinson et al., 2008).

The Q-factor can be estimated both in time and frequency domains. Nevertheless, the time-domain methods (Gladwin and Stachy, 1974; Brzostowski and McMechan, 1992) are seldom used, because they use the amplitude information of seismic data that is easily influenced by scattering, geometric spreading and other factors (Gao et al., 2011). The methods in the frequency domain include the logarithm-spectral-ratio (LSR) method (Hauge, 1981; Stainsby and Worthington, 1985), the centroid-frequency-shift (CFS) method (Quan and Harris, 1997) and the peak-frequency-shift (PFS) method (Zhang and Ulrych, 2002). Although they are much more popular than the time-domain methods (Tu and Lu, 2010; Nunes et al., 2011), their applications are still limited. The LSR method has weak stability due to the frequency bandwidth chosen to fit a slope (Nunes et al., 2011), while the CFS and the PFS method achieve good results only under the conditions of particular types of source wavelet.

Li and Liu (2015) proposed the FWE method for Q-factor estimates by applying the frequency-weighted-exponential (FWE) function to fit the amplitude spectra of seismic wavelets. Compared to the other commonly used methods the FWE method applies to various source wavelet types and has better tolerance to random noise. In this paper we first study the theories of the FWE method. Results indicate that the FWE method has the potential to perform better for the Q-factor estimates. Thus, we improve upon the current FWE method, which enhances the accuracy of the Q-factor estimates, and it also maintains the advantages of good applicability and tolerance to random noise of the original FWE method.

2. The FWE method for Q-factor estimates

The FWE method for Q-factor estimates was proposed by Li and Liu (2015) based on the frequency-weighted-exponential (FWE) function. The form of the FWE function is:

\[ W(f) = Af^n \exp \left( -\frac{f}{f_b} \right) \]  

(1)

where \( A \) is a constant for amplitude scaling; \( n \) is the symmetry index and \( f_b \) is the bandwidth factor. When a source wavelet whose shape of amplitude spectrum is in the shape of the FWE function propagates in a medium with a Q-factor for \( t \) seconds, the amplitude spectrum of the...
The attenuated wavelet can be written as:

\[ W_a(f, t) = G(t)Wf(t)e^{-\frac{\pi ft}{Q}} \]

where \( G(t) \) is a frequency independent factor. In Eq. (2) the attenuated wavelet has the same symmetry index as the source wavelet. The bandwidth factor is reduced by:

\[ f_b^s = \frac{1}{Q \cdot f_b} \quad (3) \]

Hence, the Q-factor of the medium can be estimated based on the bandwidth factor downshift between the source and attenuated wavelet:

\[ Q = \frac{\pi f_s f_b^s}{f_b^s - f_b^0} \quad (4) \]

When a source wavelet whose shape of amplitude spectrum could be compatible with the shape of the FWE function propagates in attenuating media, Eq. (4) is implemented to estimate the Q-factor by fitting the FWE function to the amplitude spectra of the source and attenuated wavelet. The symmetry indexes and bandwidth factors of the source and attenuated wavelet are obtained by calculating the centroid frequencies and variances of their amplitude spectra (Li and Liu, 2015):

\[
\begin{align*}
  n &= \frac{f^2}{\sigma^2} - 1 \\
  f_b &= \frac{\sigma^2}{f_c}
\end{align*}
\]

where \( f_c \) and \( \sigma^2 \) refer to the centroid frequency and variance of amplitude spectrum, respectively.

To estimate the Q-factor with the bandwidth factors it is crucial to have the symmetry index remains constant. To ensure the symmetry index remains constant for a source wavelet whose shape of amplitude spectrum is in the shape of non-standard FWE function, Li and Liu (2015) average the symmetry indexes of the source and attenuated wavelet calculated by Eq. (5), and recalculate the bandwidth factors by using the average as:

\[
\begin{align*}
  f_b &= \frac{f_c}{(n + 1)} \\
  f_b^0 &= \frac{f_c}{(n + 1)}
\end{align*}
\]

where \( n \) and \( n^a \) are the symmetry indexes of the source and attenuated wavelet, respectively; \( f_b^a \) and \( f_b^0 \) denote the recalculated bandwidth factors; \( f_c \) and \( f_c^a \) refer to the centroid frequencies. Therefore, Eq. (4) is modified by replacing the bandwidth factors with the recomputed ones:

\[ Q = \frac{\pi f_s f_b^s}{f_b^s - f_b^0} \quad (7) \]

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### Table 1
Attenuation test.

<table>
<thead>
<tr>
<th>Trial</th>
<th>Trial 1</th>
<th>Trial 2</th>
<th>Trial 3</th>
<th>Trial 4</th>
<th>...</th>
<th>Trial 20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source wavelet</td>
<td>The FWE function with ( n = 2 ) and ( f_b = 15 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Input Q-factor</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Travel time (ms)</td>
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<td>200</td>
<td>300</td>
<td>400</td>
<td>...</td>
<td>2000</td>
</tr>
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</table>

### Table 2
The symmetry index and bandwidth factor of source and attenuated wavelet.

<table>
<thead>
<tr>
<th>Row</th>
<th>( n^a )</th>
<th>Trial 1</th>
<th>Trial 2</th>
<th>Trial 3</th>
<th>Trial 4</th>
<th>...</th>
<th>Trial 20</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>...</td>
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<tr>
<td>4</td>
<td>15</td>
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<td>15</td>
<td>15</td>
<td>15</td>
<td>...</td>
<td>15</td>
</tr>
</tbody>
</table>

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**Fig. 1.** Source wavelet with the shape of amplitude spectrum in a shape of the FWE function.

**Fig. 2.** Source wavelet with the shape of amplitude spectrum in a shape of non-standard FWE function.
3. Improvement to the FWE method

The FWE method for Q-factor estimates replaces the amplitude spectra of source and attenuated wavelet by two modeled FWE functions with the same symmetry index. Therefore, the accuracy of Q-factor estimates depends on the similarity between the modeled FWE functions and the amplitude spectra of source and attenuated wavelet. The more the modeled FWE functions replicate the amplitude spectra of source and attenuated wavelet, the more accurate the Q-factor estimates. However, the FWE functions modeled by measuring the symmetry indexes of source and attenuated wavelet are not necessarily the best choices. Although averaging the symmetry indexes does not necessarily make the FWE functions best match the amplitude spectra of source and attenuated wavelet, the average of the symmetry indexes remains constant, the average of the symmetry indexes does not necessarily make the FWE functions best match the amplitude spectra of source and attenuated wavelet.

To further enhance the accuracy of Q-factor estimates by the FWE method, an improvement is implemented, where two FWE functions are employed to obtain the optimal symmetry index and bandwidth factor from Eqs. (5) and (6). Hence, the FWE method for Q-factor estimates replaces the amplitude spectra of source and attenuated wavelet, the more accurate the Q-factor estimates depend on the similarity between the modeled FWE functions and the amplitude spectra of source and attenuated wavelet.

From the least squares fit, the peak amplitude of the FWE function is calculated with:

\[ a_p = A \cdot \left( \frac{n - f_b}{\epsilon} \right)^n \]  

Then we obtain the amplitude-normalized FWE function:

\[ W_n(f) = \frac{f^n \exp\left(-\frac{f}{f_b}\right)}{\left(\frac{n - f}{\epsilon}\right)^n} \]  

which results in the objective function of the least squares fit:

\[ M = \sum_i \left[ \ln S_i(f_i) - \left\{ \ln \left[f_i^n \exp\left(-\frac{f_i}{f_b}\right)\right] - \ln \left(\frac{n - f_i}{\epsilon}\right)^n \right\} \right] \]  

\[ + \sum_i \left[ \ln R_i(f_i) - \left\{ \ln \left[f_i^n \exp\left(-\frac{f_i}{f_b}\right)\right] - \ln \left(\frac{n - f_i}{\epsilon}\right)^n \right\} \right]^2 \]  

where \( f_i \) is the frequency; \( S_i(f_i) \) and \( R_i(f_i) \) refer to the normalized amplitude spectra of source and attenuated wavelet, respectively. \( n, f_b \) and \( f_b^\prime \) are unknowns. This can be further simplified as:

\[ M = \sum_i \left[ \ln S_i(f_i) - \left( n \ln f_i - f_i f_b - n \ln n - n \ln f_b + n \right) \right]^2 + \]  

\[ + \sum_i \left[ \ln R_i(f_i) - \left( n \ln f_i - f_i f_b^\prime - n \ln n - n \ln f_b^\prime + n \right) \right]^2 \]  

When the objective function is minimized, the optimal \( n, f_b \) and \( f_b^\prime \) can be solved by iterations of Gradient Descent method with the initial values of \( n, f_b \) and \( f_b^\prime \) measured from Eqs. (5) and (6). Hence, the FWE

![Q-factor estimates](image)

![Relative error](image)

Fig. 3. Q-factor comparison.

Table 3

The centroid frequency and variance of source and attenuated wavelet.

<table>
<thead>
<tr>
<th>Row</th>
<th>( n )</th>
<th>( f_b )</th>
<th>( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.8209</td>
<td>3.5807</td>
<td>3.5409</td>
</tr>
<tr>
<td>2</td>
<td>5.7521</td>
<td>3.6181</td>
<td>3.5377</td>
</tr>
<tr>
<td>3</td>
<td>5.6848</td>
<td>3.5804</td>
<td>3.5406</td>
</tr>
<tr>
<td>4</td>
<td>5.6191</td>
<td>3.6169</td>
<td>3.5461</td>
</tr>
<tr>
<td>5</td>
<td>5.6965</td>
<td>3.6561</td>
<td>3.5684</td>
</tr>
<tr>
<td>6</td>
<td>5.6191</td>
<td>3.6169</td>
<td>3.5377</td>
</tr>
</tbody>
</table>
The attenuation tests of Li and Liu (2015) guided a similar test to validate Eq. (11) (Table 1). A source wavelet whose shape of amplitude spectrum is in a shape of the FWE function with $n = 2$ and $f_b = 15$ (Fig. 1) is chosen, and it propagates through a medium with a Q-factor of $Q = 100$ for different travel times. Each trial produces attenuated wavelets. Travel times increase from 100 ms to 2000 ms, and the step size is 100 ms.

After the attenuated wavelet is obtained for each trial, Eq. (5) is employed to calculate the $n^a$ and $f_b^a$ as shown in Rows 1–2 in Table 2. The symmetry index of the attenuated wavelet remains constant, whereas the bandwidth factor is reduced. For each trial Eq. (11) is performed to solve the optimal $\tilde{n}$, $f_b$ and $f_b^a$ that produce two FWE functions that best match the amplitude spectra of the source and attenuated wavelet (Table 2, Row 3, 4 and 5). Compared to the measurements in Rows 1–2 (Table 2), the optimal solutions based on the least squares method are in good agreement with the measurements by their definitions (Eq. (5)) in every trial. Meanwhile, for each trial we can produce the same optimal solutions if different values around $n$, $f_b$ and $f_b^a$ are employed as the initial values of the iteration. These results validate Eq. (11), which lays the foundation for its application to the source wavelets whose shape of amplitude spectra is in a shape of the non-standard FWE function.

Fig. 2 shows a filtered amplitude spectrum that is determined by the Butterworth filter. It is employed as the amplitude spectrum for the new source wavelet to repeat the test. We fit the new spectrum with the FWE function using Eq. (5) and obtain $n = 5.8912$ and $f_b = 3.5439$.

Similarly, for each trial $n^a$, $f_b^a$ and $f_b^a$ are measured with Eqs. (5) and (6), as shown in Rows 1–3 in Table 3, and a least squares fit is conducted with the initial values of $n$, $f_b^a$ and $f_b^a$ to obtain the optimal $\tilde{n}$, $f_b$ and $f_b^a$ shown in the last three rows in Table 3.

$Q = \frac{\pi f_b^2}{f_b - f_b^a}$

(12)
The symmetry index and bandwidth factor obtained for each trial with the least squares fit are different from those defined by Eqs. (5) and (6). The differences are small at short travel times of the source wavelet, however they increase with increasing travel times. Such results indicate the FWE functions modeled with measuring the symmetry index and bandwidth factor by their definitions are indeed not the best match for the amplitude spectra of the source and attenuated wavelet. Hence, it is necessary to employ two FWE functions to fit the amplitude spectra of the source and attenuated wavelet with the Least Square Method to obtain the optimal symmetry index and bandwidth factor.

Next we compare the accuracy of Q-factor estimates from the FWE method to the improved FWE method. Measurements are taken from Table 3 to calculate the Q-factor for each trial with Eq. (7) and Eq. (12), respectively, as shown in Fig. 3a. Fig. 3b illustrates the relative errors of the Q-factor estimates between the two methods. Results show that the accuracy of the Q-factor estimates with the improved FWE method is enhanced clearly, particularly at long travel times. The relative errors from all trials are <1%. These results indicate the FWE method for Q-factor estimates is well improved when two FWE functions are applied to fit the amplitude spectra of the source and attenuated wavelet with the Least Square Method to obtain the optimal symmetry index and bandwidth factor.

5. Comprehensive comparison

We conducted three additional attenuation tests to do further comparisons of the FWE method before and after the improvement. We change the source wavelet in Table 1 to a Gaussian wavelet with $f_c = 40$ Hz and $\sigma^2 = 120$ (Fig. 4a), a Ricker wavelet with peak frequency $f_p = 40$ Hz (Fig. 5a), and a real wavelet from seismic data (Fig. 6a),
respectively. For each test zero-mean Gaussian random noise with 10% of wavelet energy was added to both the source and attenuated wavelet in every trial before determining the Q-factor estimates. The relative errors of the Q-factor estimates from the two methods are shown in Fig. 5b, 6b and 7b. Results indicate the improved FWE method is more accurate than the original FWE method in all three cases which include theoretical and real source wavelets. It also shows stability for the noisy datasets. Hence, the improved FWE method not only enhances the accuracy of the Q-factor estimates, but also maintains the advantages of good applicability and tolerance to random noise of the original FWE method.

6. Case study

Fig. 7 is zero-offset VSP records acquired in a well located in East China. There are six-level records with 312 traces at 10 m intervals. The depth range of the measurements was 200 m–3310 m, which was drilled through 19.3 m gas-bearing sandstone with a target depth of 2880–3030 m. We use these records to validate the improved FWE method for actual dataset.

Before implementation of the Q-factor estimates, source signature deconvolution filters are designed and applied to each VSP trace along with the source wavelets for all shots recorded by a geophone near the well to satisfy the assumption of source consistency (Hauge, 1981; Stainsby and Worthington, 1985). Then we apply median filter to separate the upgoing and downgoing wavefield (Raoult et al., 1984) to obtain the downgoing wave that favors the Q-factor estimates (Hauge, 1981; Stainsby and Worthington, 1985; Amundsen and Mittet, 1994). Additionally some denoising (e.g., tube waves, random noise) was conducted to obtain the results.

The downgoing traces around the target layer are shown in Fig. 8, where the gray shaded area illustrates the location of the target layer. We divide the downgoing traces into, on average, 5 layers with a thickness of 50 m each (Fig. 8), where Layer IV is the target layer. For each layer two FWE functions are employed to fit the incoming and outgoing direct downgoing wave with the Least Square Method to obtain the optimal symmetry index and bandwidth factor. Therefore, the Q-factor is estimated by the improved FWE method. The estimated interval Q-factors of the 5 layers are shown in Fig. 9. For comparisons, we also perform the original FWE method to estimate the Q-factors for the 5 intervals.

In Fig. 9, both the Q-factors estimated from the improved FWE and the original FWE methods are clearly low at the target layer, which is in good agreement with the absorption characteristic of gas-bearing sandstone. In other layers, the Q-factors estimated from the improved FWE method are relatively stable, whereas there are appreciable drops in the estimates from the original FWE method. In addition, the contrast of the Q-factors estimated from the improved FWE method between the target layer and non-target layer is more prominent than that by the original FWE method. Hence, the Q-factors estimated by the improved FWE method better represent the layer attributes and favor the indication of gas reservoirs.

7. Conclusions

In the original FWE method for Q-factor estimates, the FWE functions modeled with measurements of the symmetry index and bandwidth factor by their definitions are not the best match for the amplitude spectra of source and attenuated wavelet. This decreases the accuracy of Q-factor estimates. Here we apply two FWE functions to fit the amplitude spectra of source and attenuated wavelet with the Least Square Method to obtain the optimal symmetry index and bandwidth factor, which proves to be an effective improvement to the original FWE method. The improved FWE method enhances the accuracy of Q-factor estimates, and it also maintains the advantages of good applicability and tolerance to random noise of the original FWE method.

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