A new method for interval $Q$-factor inversion from seismic reflection data

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**ABSTRACT**

The frequency-weighted-exponential (FWE) function was designed to fit various asymmetric amplitude spectra by two parameters: symmetry index and bandwidth factor. A relation can be built between the FWE function and the $Q$-factor of media. When a source wavelet propagates in attenuating media, the FWE function is used to fit the amplitude spectra of the source and attenuated wavelet to obtain the symmetry indexes and bandwidth factors. After recalculating the bandwidth factors based on the average of the symmetry indexes, the $Q$-factor can be estimated by the decrease in the bandwidth factor between the source and attenuated wavelet. Compared with the existing commonly used methods, this method applies to various source wavelet types, and it has better tolerance to random noise. On this basis, we further evaluated an approach to invert the interval $Q$-factor from seismic reflection data. We used the FWE function to fit the local spectral contents of the interval seismic reflection data to obtain the symmetry indexes and bandwidth factors. After recalculating the bandwidth factors with the average of the symmetry indexes, the reciprocal of the bandwidth factor versus traveltime was linearly fit, and then the $Q$-factor can be calculated from the slope. This approach has high accuracy without having to correct for the tuning effect when the attenuation is high. It is suitable for different types of synthetic seismic traces and also produces good results in a field-data case study.

**INTRODUCTION**

The $Q$-factor is used to quantify the attenuation effect of seismic waves when they propagate through the earth due to the absorption of media (Ricker, 1953; Futterman, 1962; Kneib and Shapiro, 1995). It is a useful tool for hydrocarbon detection and reservoir characterization (Korneev et al., 2004; Li et al., 2006; Pinson et al., 2008) and can also be used to recover the frequency components of seismic waves by an inverse $Q$ filter (Zhang and Ulrych, 2002; Wang, 2004).

Among the existing methods for $Q$-factor estimates, frequency-domain methods are commonly used (Tu and Lu, 2010; Nunes et al., 2011), including the logarithmic spectral ratio (LSR) method (Hauge, 1981; Stainsby and Worthington, 1985), the centroid frequency shift (CFS) method (Quan and Harris, 1997), and the peak frequency shift (PFS) method (Zhang and Ulrych, 2002). Nevertheless, the LSR method is easily influenced by the frequency bandwidth chosen to fit a slope, and therefore, it is difficult to parameterize (Nunes et al., 2011). The CFS and PFS methods were proposed under the assumption of particular source wavelet types. They both have poor applicability. (Tu and Lu, 2010; Nunes et al., 2011). Hu et al. (2011) design the frequency-weighted-exponential (FWE) function, which can be used to fit various asymmetric amplitude spectra by two separate parameters: the symmetry index and the bandwidth factor, and they introduce it into attenuation tomography. Based on this, we have studied the relation between the FWE function and the $Q$-factor of media, and then we proposed a new method for $Q$-factor estimates by using the symmetry index and the bandwidth factor. Compared with existing commonly used methods, our method applies to various source wavelet types and has better tolerance to random noise.

Attenuation is often calculated from seismic transmission data, such as vertical seismic profile data (Hauge, 1981; Stainsby and Worthington, 1985; Amundsen and Mittet, 1994), crosswell data (Neep et al., 1996; Quan and Harris, 1997), and sonic logging data (Sun et al., 2000). Seismic reflection data can also be used for $Q$-factor estimates (Tonn, 1991; Dasgupta and Clark, 1998; Hackert and Parra, 2004; Reine et al., 2012a, 2012b), but they are inevitably influenced by the tuning effect that alters the shape of the spectra used to calculate the $Q$-factor (Hackert and Parra, 2004). To correct
the tuning effect, Hackert and Parra (2004) use nearby well-log impedance data to eliminate reflection coefficients. Tu and Lu (2010) present a spectral correction method by estimating wavelets based on higher order statistics. However, well-log data are not always available and the accuracy of wavelet estimation is difficult to guarantee. Thus, the application of the two methods is limited. Based on our new method for Q-factor estimates, we further propose an approach to invert the interval Q-factor from seismic reflection data by using the symmetry indexes and bandwidth factors of local spectral contents. Our approach has good accuracy and applicability without having to correct for the tuning effect when the attenuation is high. Its application in a field-data case study produces good results.

A NEW METHOD FOR Q-FACTOR ESTIMATES

The frequency-weighted-exponential function

The FWE function (Hu et al., 2011) was designed to fit various asymmetric amplitude spectra. The form of the FWE function is

\[ W(f) = A f^n \exp \left( -\frac{f}{f_b} \right), \] (1)

where \( A \) is a constant for amplitude scaling; \( n \) is the symmetry index controlling the symmetry property, and \( f_b \) is the bandwidth factor controlling the bandwidth. The centroid frequency of \( W(f) \) can be calculated as

\[ f_c = \frac{\int_0^\infty fW(f)df}{\int_0^\infty W(f)df} = (n+1)f_b, \] (2)

and the variance is

\[ \sigma^2 = \frac{\int_0^\infty (f-f_c)^2W(f)df}{\int_0^\infty W(f)df} = (n+1)f_b^2. \] (3)

![Figure 1. Amplitude-spectrum fitting by the FWE function.](image)

The centroid frequency and variance of the FWE function have very simple forms composed of \( n \) and \( f_b \). This makes it easy to fit the asymmetric amplitude spectra by using the FWE function because \( n \) and \( f_b \) can be obtained by calculating the centroid frequency and variance of the amplitude spectra

\[
\begin{align*}
\{ n &= \frac{\sigma^2}{\sigma_0^2} - 1, \\
f_b &= \frac{\sigma_0^2}{f_c}.
\end{align*}
\] (4)

Figure 1 illustrates three examples in which the FWE function is used to fit the amplitude spectra of seismic wavelets. Figure 1a is a Gaussian wavelet with \( f_c = 60 \) Hz and \( \sigma_0^2 = 150 \). According to equation 4, its amplitude spectrum can be fit by the FWE function with \( n = 23 \) and \( f_b = 2.5 \), as shown in Figure 1b. Figure 1c is a Ricker wavelet with peak frequency \( f_p = 60 \) Hz. By calculating its centroid frequency and variance, the FWE function that fits its amplitude spectrum has \( n = 4.6 \) and \( f_b = 13.58 \), as shown in Figure 1d. Figure 1e is a real wavelet from seismic data. Its amplitude spectrum can be fit by the FWE function with \( n = 3.79 \) and \( f_b = 17.08 \), as shown in Figure 1f. From these three examples, we can see that because the spectra of the theoretical wavelets are smooth and simple, the fitted FWE functions almost overlap them, whereas for the real wavelet with a much more complicated spectrum, the FWE function behaves like the trend line of its spectrum.

Q-factor estimates

When a source wavelet whose amplitude spectrum’s shape is in the shape of the FWE function propagates in a medium with a Q-factor for \( t \) s, according to the Aki-Richards theory (Aki and Richards, 2002), the amplitude spectrum of the attenuated wavelet can be written as

\[
W^a(f, t) = G(t)W(f)e^{-\pi t f^2_Q f_b^2}.
\]

Where

\[
G(t) = G(r)A f^n \exp \left( -f \left( \frac{\pi t}{Q} + \frac{1}{f_b} \right) \right).
\] (5)

where \( G(t) \) is a frequency-independent factor including the effects of geometrical spreading, reflection/transmission coefficients, etc. (Červený, 2001). From equation 5, we can see that the amplitude spectrum of the attenuated wavelet still maintains the form of the FWE function. Compared with the source wavelet, the symmetry index remains constant during the propagation, whereas the bandwidth factor of the attenuated wavelet is altered as

\[
f_b^a = \frac{1}{Q} f_b + \frac{1}{f_b}.
\] (6)

The variation in the bandwidth factor between the source and attenuated wavelet is associated with the Q-factor of the medium and the travel-time of the source wavelet. Hence, we can estimate the Q-factor by the decrease in the bandwidth factor between the source and attenuated wavelet given as
\[
Q = \frac{\pi tf_b f_a^2}{f_b - f_a^2}.
\]

Because the FWE function can be used to fit the amplitude spectra of seismic wavelets, when a source wavelet whose amplitude spectrum’s shape could be compatible with the shape of the FWE function propagates in attenuating media, we can use the FWE function to fit the amplitude spectra of the source and attenuated wavelet to obtain the symmetry indexes and bandwidth factors, and then we estimate the \(Q\)-factor by using equation 7. However, according to equation 5, that the symmetry index remains constant is the prerequisite for estimating the \(Q\)-factor by using the bandwidth factors. This condition is probably difficult to satisfy for a source wavelet whose amplitude spectrum’s shape is in the shape of the nonstandard FWE function. To solve this problem, we can average the symmetry indexes of the source and attenuated wavelet, and then we recalculate the bandwidth factors by using the average and the centroid frequencies according to equation 2, as shown in the following equation:

\[
\begin{cases}
\bar{n} = (n + n^a)/2, \\
f_a = f_c / (\bar{n} + 1), \\
f_a^a = f_c^a / (\bar{n} + 1),
\end{cases}
\]

where \(n\) and \(n^a\) are the symmetry indexes of the source and attenuated wavelet, respectively; \(f_a^b\) and \(f_a^a\) denote the recalculated bandwidth factors; and \(f_c\) and \(f_c^a\) are the centroid frequencies. In this way, the symmetry index of the source and attenuated wavelet remains a constant. Thus, in equation 7, we replace the bandwidth factors by the recomputed ones, and we can get a modified equation:

\[
\bar{Q} = \frac{\pi tf_b f_a^2}{f_b - f_a^2}.
\]

By using equations 8 and 9, the \(Q\)-factor can be estimated for the source wavelet whose amplitude spectrum’s shape could be compatible with the shape of the FWE function.

Furthermore, when we substitute equation 4 and 8 into equation 9, we obtain

\[
\bar{Q} = \frac{2}{\frac{f_c^2}{\sigma_a^2} + \frac{f_c^2}{\sigma_c^2} + \frac{f_c^2}{\sigma_a^2 f_c}},
\]

where \(f_c\) and \(f_c^a\) denote the centroid frequencies of the source and attenuated wavelet, respectively, and \(\sigma_a^2\) and \(\sigma_c^2\) refer to the variances. Compared with equation 7, a relation between the \(Q\)-factor estimates before and after the modification is obtained as

\[
\bar{Q} = \frac{1}{\frac{1}{2} \left( Q + \bar{Q}_{corr} \right)},
\]

where

\[
\bar{Q}_{corr} = \frac{1}{\frac{f_c^2}{\sigma_a^2} + \frac{f_c^2}{\sigma_c^2} + \frac{f_c^2}{\sigma_a^2 f_c}}.
\]

It is easy to prove that for a source wavelet whose amplitude spectrum’s shape is in the shape of the FWE function, the \(\bar{Q}_{corr}\) is equal to \(Q\), so the new estimate \(\bar{Q}\) is equal to \(Q\) according to equation 11. Thus, the \(\bar{Q}_{corr}\) can be considered as the correction term made by the step of averaging the symmetry indexes especially for the source wavelet whose amplitude spectrum’s shape is in the shape of the nonstandard FWE function.

**Validity analysis**

Table 1 illustrates a test of the above theoretical analysis. We choose a source wavelet whose amplitude spectrum’s shape is in a shape of the FWE function with \(n = 1\) and \(f_b = 25\) (Figure 2a), and we make it propagate in a medium with a \(Q\)-factor of \(Q = 100\) for different traveltimes in different trials to produce the attenuated wavelets. The traveltime gradually increases from 100 to 2000 ms, and the step size is equal to 100 ms.

For each trial, after the attenuated wavelet is obtained, we fit its spectrum by using the FWE function. According to equation 4, the symmetry index and bandwidth factor of each attenuated wavelet are obtained by calculating the centroid frequency and variance, as is shown in Table 1. The symmetry index of the attenuated wavelet remains constant, whereas the bandwidth factor is reduced with the increasing traveltime.

For each trial, by using equation 7, we estimate the \(Q\)-factor of the medium at different traveltimes. Figure 2b shows a comparison between the estimated \(Q\)-factors and the input ones. At different traveltimes, they are in good agreement. The exact values of the estimated \(Q\)-factors are shown in the last row in Table 1.

The test proves the validity of the theoretical analysis when the shape of the amplitude spectrum of the source wavelet is in the shape of the FWE function. Next, we alter it into a shape of the nonstandard FWE function to repeat the test. As shown in Figure 3a.

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**Table 1. Attenuation test when the shape of the amplitude spectrum of the source wavelet is in the shape of the frequency-weighted-exponential function.**

<table>
<thead>
<tr>
<th>Traveltime (ms)</th>
<th>—</th>
<th>Trial 1</th>
<th>Trial 2</th>
<th>Trial 3</th>
<th>Trial 4</th>
<th>...</th>
<th>Trial 20</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>...</td>
<td>1.0000</td>
</tr>
<tr>
<td>(f_b)</td>
<td>25.0000</td>
<td>23.1794</td>
<td>21.6060</td>
<td>20.2327</td>
<td>19.0235</td>
<td>...</td>
<td>9.7244</td>
</tr>
<tr>
<td>Input (Q)-factor</td>
<td>—</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>...</td>
<td>100</td>
</tr>
<tr>
<td>Estimated (Q)-factor</td>
<td>—</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>...</td>
<td>100</td>
</tr>
</tbody>
</table>
we filter the FWE function (the same as in Figure 2a) by a Butterworth filter, and we use the filtered function as the amplitude spectrum of the new source wavelet. Because the shape of the new spectrum is not in the shape of the standard FWE function, we have to refit it by the FWE function and obtain $n = 2.5978$ and $f_b = 6.0487$.

As shown in Table 2, for each trial after we get the attenuated wavelet, we fit its spectrum, obtaining the symmetry index and bandwidth factor. The bandwidth factor of the attenuated wavelet is also observed to decrease with the increasing traveltime. However, the symmetry indexes of the attenuated wavelets in every trial are quite different from that of the source wavelet. The variation in the symmetry index between the source and attenuated wavelet increases with the increase in traveltime. If we neglect this problem and directly use the bandwidth factors in Table 2 to estimate the $Q$-factors at different traveltimes by equation 7, we get the estimates in Figure 3b. They show large errors compared with the correct values. The errors increase with the increase in traveltime. This indicates that the larger the variation in the symmetry index between the source and attenuated wavelet, the poorer is the accuracy of the $Q$-factor estimates.

As in the previous theoretical analysis, to solve this problem, in each trial, we average the symmetry indexes of the source and attenuated wavelet, and then we recalculate the bandwidth factors according to equation 8. By using equation 9, we estimate the $Q$-factors again as shown in Figure 3b. The accuracy of the new $Q$-factor estimates improves significantly compared with that of the old ones. This test proves that to maintain the symmetry index constant is the prerequisite for estimating the $Q$-factor by using the bandwidth factors. Recalculating the bandwidth factors based on the average of the symmetry indexes is an essential processing step when the shape of the amplitude spectrum of the source wavelet is in the shape of the nonstandard FWE function. The exact values of the estimated $Q$-factors before and after recalculating the bandwidth factors are shown in the last two rows in Table 2.

**Applicability analysis**

Because the FWE function can fit the amplitude spectra of the seismic wavelets, the method for $Q$-factor estimates based on the...
The FWE function (hereafter referred to as the FWE method) should be able to be used to estimate the $Q$-factor for various source wavelet types. To verify its applicability, we perform additional attenuation tests by changing the source wavelet into a Gaussian wavelet with $f_c = 60$ Hz and $\sigma^2 = 150$ (Figure 1a), a Ricker wavelet with peak frequency $f_p = 60$ Hz (Figure 1c), and a real wavelet from seismic data (Figure 1e), respectively.

In addition to estimating the $Q$-factor by using the FWE method, the LSR, CFS, and PFS methods are also used for comparison. Moreover, before the estimates, zero-mean Gaussian random noise with 10% wavelet energy is added to the source and attenuated wavelet in every trial to further compare the tolerance of every method to random noise.

Take the test of the Gaussian source wavelet for example. In the FWE and CFS methods, the centroid frequency and variance need be calculated. Their calculations are strongly influenced by the noise at low and high frequencies. Because we add the noise of the same energy to the source and attenuated wavelets in every trial, the amplitude spectra of all the source and attenuated wavelets have the same noise level, as shown in Figure 4. Hence, we define a threshold that is above the noise at low and high frequencies, as shown in the dashed lines in Figure 4, so that we can use the spectrum value above the threshold to calculate the centroid frequencies and variances of the source and attenuated wavelet in each trial.

In the LSR method, as shown in Figure 5, the LSR of the source and attenuated wavelet is also influenced by the noise at low and high frequencies. We have to use the LSR at dominant frequencies in which they have a relatively high signal-to-noise ratio to fit a slope. However, in all 20 trials, it is impractical to pick the dominant bandwidth one by one. According to Figure 5, we decide to choose the bandwidth of 50–70 Hz (as shown in the dashed rectangles in Figure 5) to fit the slopes in which the LSR of all the trials have a relatively high S/N.

Figure 6 shows the $Q$-factor estimates of the Gaussian source wavelet case. To clearly distinguish every method, the logarithmic coordinate is used to show the relative errors of the estimates. As we can see from the figure, the FWE method is the most accurate among all of the methods. The apparent improvement in accuracy of the FWE method at approximately $t = 1.2$ s is probably connected with the effect of the noise on the calculation of the centroid frequency and variance. Although the CFS method was proposed on the assumption that the source wavelet is of Gaussian shape (Quan and Harris, 1997) and the LSR method is not restricted by the source wavelet type, they still present slightly less tolerance to random noise than the FWE method. The PFS method shows huge errors due to its inapplicability to non-Ricker wavelets (Tu and Lu, 2010).

Figure 7 illustrates the relative errors of the $Q$-factor estimates of the Ricker source wavelet case. For this case, the FWE method is still the most accurate. The LSR method ranks second. The CFS method shows large errors at long traveltimes due to its inapplicability to non-Gaussian wavelets. Although the PFS method was proposed on the assumption that the source wavelet is a Ricker wavelet, it presents large fluctuations because the position of the peak frequency is easily influenced by the noise.

Figure 8 illustrates the attenuation test of the real wavelet. Although the amplitude spectrum of the real wavelet is more complicated than that of the theoretical wavelet, as shown in Figure 1, the FWE method still presents the highest accuracy among these four methods.

In conclusion, compared with the existing commonly used methods, the FWE method is able to be used to estimate the $Q$-factor for various source wavelet types, and it shows better tolerance to random noise.

In addition, it is worth noting that the measurements of the centroid frequency and variance are crucial to the FWE method. For the theoretical wavelets without noise, the amplitude spectra at a whole bandwidth can be used to calculate the centroid frequency and variance, whereas for the data with noise, especially the real data, picking the effective spectrum value by defining the threshold is extremely important for the measurements.

### Table 2. Attenuation test when the shape of the amplitude spectrum of the source wavelet is in the shape of the nonstandard frequency-weighted-exponential function.

<table>
<thead>
<tr>
<th>Traveltime (ms)</th>
<th>Trial 1</th>
<th>Trial 2</th>
<th>Trial 3</th>
<th>Trial 4</th>
<th>...</th>
<th>Trial 20</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>2.5978</td>
<td>2.5950</td>
<td>2.5863</td>
<td>2.5728</td>
<td>2.5550</td>
<td>...</td>
</tr>
<tr>
<td>$f_b$</td>
<td>6.0487</td>
<td>5.4991</td>
<td>5.4187</td>
<td>5.3480</td>
<td>5.2857</td>
<td>...</td>
</tr>
<tr>
<td>Input $Q$-factor</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>...</td>
</tr>
<tr>
<td>Estimated $Q$-factor before recalculating the bandwidth factors</td>
<td>104.897</td>
<td>109.027</td>
<td>112.041</td>
<td>114.749</td>
<td>...</td>
<td>147.427</td>
</tr>
<tr>
<td>Estimated $Q$-factor after recalculating the bandwidth factors</td>
<td>100.015</td>
<td>100.054</td>
<td>100.112</td>
<td>100.184</td>
<td>...</td>
<td>101.139</td>
</tr>
</tbody>
</table>

### INTERVAL $Q$-FACTOR INVERSION FROM SEISMIC REFLECTION DATA

#### Theory

When a source wavelet propagates in a medium with a $Q$-factor, the bandwidth factor of the attenuated wavelet is altered gradually with the increasing traveltime, as shown in equation 6. We change the form of equation 6 into

$$\frac{1}{f_b} = \frac{n t}{Q} + \frac{1}{f_b}. \quad (13)$$

Equation 13 is a linear representation between the reciprocal of the bandwidth factor and the traveltime, and it can be written as
\[ Y = At + B, \]  

where \( Y = 1/f_b^a \), \( A = \pi/Q \), and \( B = 1/f_b \); \( A \) is the slope and \( B \) is the intercept. Hence, if we get the bandwidth factor of the attenuated wavelet versus traveltime, we can build an overdetermined equation by equation 14 as

\[
\begin{bmatrix}
1/f_b^a \\
1/f_b^a \\
1/f_b^a \\
\vdots \\
1/f_b^a 
\end{bmatrix}
\begin{bmatrix}
t_1 \\
t_2 \\
t_3 \\
\vdots \\
t_m 
\end{bmatrix}
= \begin{bmatrix} A \\ B \end{bmatrix},
\]

(15)

Figure 4. Amplitude spectra with noise: (a) source wavelet, (b) attenuated wavelet in trial 10, and (c) attenuated wavelet in trial 20.

Figure 5. The LSR of the source and attenuated wavelet versus frequency: (a) trials 1, (b) 10, and (c) 20.

Figure 6. Relative errors of \( Q \)-factor estimates of the Gaussian source wavelet case.

Figure 7. Relative errors of \( Q \)-factor estimates of the Ricker source wavelet case.
where $A$ and $B$ are unknowns, $m$ is the number of time samples, and $f_{bk}^w(k = 1, 2, \ldots, m)$ denotes the bandwidth factor of the attenuated wavelet at $t_k(k = 1, 2, \ldots, m)$; $A$ and $B$ can be solved by least-squares inversion, and therefore, we easily get the $Q$-factor from the slope $A$:

$$Q = \frac{\pi}{A}. \quad (16)$$

However, for reflection seismic data, we have to take reflectivity sequences into account. Because of the tuning effect, the bandwidth factors of the attenuated wavelets cannot be easily obtained.

To study the $Q$-factor inversion from seismic reflection data, first we simulate an attenuated synthetic seismic trace. Assume that a source wavelet $w$ propagates in a medium with a $Q$-factor for $t$; $w_1, w_2, \ldots, w_m$ ($m$ is the sampling number of $t$) are the attenuated wavelets during the propagation. According to the matrix form of the convolution model, the attenuated synthetic seismic trace $s$ is generated as (Tu and Lu, 2010)

$$s = \begin{bmatrix}
  w_{11} & 0 & 0 & \cdots & 0 & 0 & 0 \\
  w_{12} & w_{21} & 0 & \cdots & 0 & 0 & 0 \\
  w_{13} & w_{22} & w_{31} & \cdots & 0 & 0 & 0 \\
  \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
  0 & 0 & 0 & \cdots & w_{m-2j} & w_{m-1j} & w_{mj} \\
  0 & 0 & 0 & \cdots & 0 & w_{m-1j} & w_{mj} \\
  0 & 0 & 0 & \cdots & 0 & 0 & w_{mj}
\end{bmatrix} \times \begin{bmatrix}
  r_1 \\
  r_2 \\
  r_3 \\
  \vdots \\
  r_{m-2} \\
  r_{m-1} \\
  r_m
\end{bmatrix}, \quad (17)$$

where $j$ is the sampling number of $w$; $w_i(i = 1, 2, \ldots, m; k = 1, 2, \ldots, j)$ denotes the $k$th element of $w(i = 1, 2, \ldots, m)$; and $[r_1, r_2, \ldots, r_m]^T$ is the reflectivity sequence. Figure 9 illustrates an example of an attenuated synthetic seismic trace. The source wavelet is a Ricker wavelet with a peak frequency of $f_p = 60$ Hz. The reflectivity is a Bernoulli-Gaussian-distributed sequence with a sparsity coefficient of 0.6. The traveltime of the source wavelet is 1 s. The $Q$-factor of the medium is 50.

In the attenuated seismic trace, the attenuated wavelets are tuned together, and therefore, it is difficult to get all the attenuated wavelets during the propagation. However, we believe that the characteristics of the attenuated wavelets are still preserved in the attenuated seismic trace to a certain extent. We add a sliding time window shifting along the sampling point onto the attenuated seismic trace to perform the short-time Fourier transform (STFT) to get the local spectral contents as shown in Figure 10. The length of the time window we used here is one-fourth of the trace length.

It is observed from Figure 10 that the bandwidth of the local spectral content decreases gradually with the increasing traveltime. Hence, we use the FWE function to fit each local spectral content in Figure 10 to get the symmetry indexes and bandwidth factors, as shown in Figure 11. To do comparisons, we also calculate the symmetry index and bandwidth factor of the attenuated wavelet versus traveltime. In Figure 11a, the symmetry indexes of the local spectral content and the attenuated wavelet have changed with the traveltime. Hence, in Figure 11b, the bandwidth factors of the local spectral contents and the attenuated wavelets have been recalculated, respectively. In Figure 11b, although the dotted curve is not as smooth as the solid one, it still shows the tendency of a decrease with the traveltime.
increasing traveltine. Moreover, the solid curve behaves like the
trend line of the dotted one. That is crucial for the
$Q$-factor in-
version.

According to equations 13 and 14, there is a linear relation be-
tween the reciprocal of the bandwidth factor of attenuated wavelet
and the traveltine. After we change the $x$-coordinate of Figure 11b
into the reciprocal of the bandwidth factor, it is observed that the
reciprocal of the bandwidth factor of the attenuated wavelets be-
haves like the linear fitting curve of that of the local spectral con-
tents, as shown in Figure 12. Hence, in equation 15, we replace
$f_{bk}^{R}$
b by $f_{bk}^{l}$, which denotes the bandwidth factor of the local spectral con-
tent at $t_{k}$. Then, we invert the slope $A = 0.0612$. By using equa-
tion 16, the $Q$-factor is calculated as 51.36, which is very close
to the input value.

**Stability analysis**

In the above analysis, the length of the time window is an im-
portant parameter in the STFT. To study its effect on the
$Q$-factor
inversion, we repeat the above process by using different lengths of
the time window, as shown in Figure 13. Be-
tween one-fifth and one-half of the trace length,
the length of the time window does not have
much of an impact to the
$Q$-factor inversion. That is because it only affects the fluctuation extent of
the bandwidth factor versus traveltime, but the
small change of the fluctuation extent has little
influence on the slope of the reciprocal of the
bandwidth factor versus traveltime. We prefer
to set it to one-fourth of the trace length because
then the STFT has the best time-frequency reso-
lution (Cohen, 1995).

Furthermore, we analyze the uncertainty of the
results because the calculation of $A$ is a linear in-
verse problem. We rewrite equation 15 as
\[
\mathbf{d} = \mathbf{Gm},
\]
where
\[
\mathbf{d} = \begin{bmatrix}
\frac{1}{f_{b_1}} \\
\frac{1}{f_{b_2}} \\
\vdots \\
\frac{1}{f_{b_m}}
\end{bmatrix},
\]
\[
\mathbf{G} = \begin{bmatrix}
t_1 & 1 \\
t_2 & 1 \\
t_3 & 1 \\
\vdots & \vdots \\
t_m & 1
\end{bmatrix},
\]
and $\mathbf{m} = [A/B]$. The general covariance matrix of $\mathbf{m}$ for a least-
squares solution is thus (Aster et al., 2013)
\[
\text{Cov}(\mathbf{m}_{L_2}) = \sigma^2(d)(\mathbf{G}^T\mathbf{G})^{-1},
\]
where $\sigma^2(d)$ is the variance of $d$. By using equation 21, the standard deviation of the slope $A$ is calculated as 0.003. Thus, the confidence interval of slope $A$ is $0.0612 \pm 0.003$, which presents good robustness. We transform it to the confidence interval of the $Q$-factor and obtain $[48.93 - 53.98]$.

Besides the uncertainty of the inversion, we further analyze the effect of the randomness of the reflectivity sequence on the results because we use a Bernoulli-Gaussian-distributed reflectivity sequence. We carry out 100 independent trials with different realizations of the reflectivity sequence. Figure 14 illustrates the histogram of the $Q$-factors inverted from the 100 trials. The mean of all the $Q$-factors is 51.09, and the standard deviation is 3.24, which still shows good stability.

From the above, we conclude that, for seismic reflection data, we can use the FWE function to fit the local spectral contents of seismic reflection data to obtain the symmetry indexes and bandwidth factors. After recalculating the bandwidth factors based on the average of the symmetry indexes, we linearly fit the reciprocal of the bandwidth factor versus traveltime, and then, the $Q$-factor can be inverted by the slope. Without having to correct for the tuning effect, this method (hereafter referred to as the FWE inversion method) has good accuracy. Meanwhile, using multiple spectra for all of the time samples to invert the $Q$-factor makes the results more stable than using only two spectra as in equation 9. As is typical of interval attenuation measurements, the FWE inversion method is most appropriate when the interval chosen contains a set of sediments with similar properties, and an approximately constant $Q$.

### Applicability analysis

Tables 3–6 summarize several tests designed to analyze the applicability of the FWE inversion method. We first examine Table 3. It illustrates four tests to analyze its applicability to different dominant frequencies of the source wavelet. Four attenuated synthetic seismic traces are simulated with the same Bernoulli-Gaussian-distributed reflectivity sequence, traveltime of the source wavelet, and $Q$-factor, except that the Ricker source wavelets have different peak frequencies, as shown in the table. We invert the $Q$-factor for each trace and carry out 100 independent trials. The mean and the standard deviation of the $Q$-factors of every trace are shown in the last row of Table 3 and in Figure 15a. At different dominant frequencies of the source wavelet, the inverted $Q$-factors have approximately the same accuracy, which indicates that the approach is not restricted by the dominant frequency of the source wavelet.

### Table 3. Test for the applicability to the dominant frequency of the source wavelet.

<table>
<thead>
<tr>
<th>Source wavelet</th>
<th>Trace 1</th>
<th>Trace 2</th>
<th>Trace 3</th>
<th>Trace 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak frequency (Hz)</td>
<td>20</td>
<td>40</td>
<td>60</td>
<td>80</td>
</tr>
<tr>
<td>Reflectivity sequence</td>
<td>Sparsity coefficient</td>
<td>0.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Traveltime (s)</td>
<td></td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Input $Q$-factor</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>Inverted $Q$-factor</td>
<td>$51.34 \pm 3.79$</td>
<td>$48.60 \pm 3.77$</td>
<td>$51.07 \pm 3.69$</td>
<td>$51.52 \pm 3.87$</td>
</tr>
</tbody>
</table>
Tables 4–6 are designed to analyze the applicability of the approach to different sparsity coefficients of the reflectivity sequence, traveltimes of the source wavelet, and S/N, respectively. Figure 15b–15d shows the results. As shown in Figure 15b, the accuracy of the approach is also not influenced by the sparsity coefficient of the reflectivity sequence. In Figure 15c, the traveltime of the source wavelet affects the inversions. The accuracy reduces with the decrease in traveltime, probably because the bandwidth factor of the local spectral content versus traveltime has poor regularity at low attenuation. This is an inevitable problem because the occurrence of attenuation needs enough time, and the higher the attenuation is, the higher the stability of the $Q$ estimates is (Nunes et al., 2011). The approach also exhibits good tolerance to random noise as shown in Figure 15d. When zero-mean Gaussian random noise with 10% wavelet energy is added to the synthetic seismic traces, the inverted $Q$-factors still have relatively high accuracy.

**Table 4. Test for the applicability to the sparsity coefficient of the reflectivity sequence.**

<table>
<thead>
<tr>
<th>Source wavelet</th>
<th>Reflectivity sequence</th>
<th>Trace 1</th>
<th>Trace 2</th>
<th>Trace 3</th>
<th>Trace 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak frequency (Hz)</td>
<td>Sparsity coefficient</td>
<td>60</td>
<td>0.2</td>
<td>0.4</td>
<td>0.6</td>
</tr>
<tr>
<td>Traveltime (s)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Input $Q$-factor</td>
<td></td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>Inverted $Q$-factor</td>
<td></td>
<td>51.21 ± 3.61</td>
<td>51.44 ± 3.60</td>
<td>51.69 ± 3.79</td>
<td>48.60 ± 3.58</td>
</tr>
</tbody>
</table>

**Table 5. Tests for the applicability to the traveltime of the source wavelet.**

<table>
<thead>
<tr>
<th>Source wavelet</th>
<th>Reflectivity sequence</th>
<th>Trace 1</th>
<th>Trace 2</th>
<th>Trace 3</th>
<th>Trace 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak frequency (Hz)</td>
<td>Sparsity coefficient</td>
<td>60</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>Traveltime (s)</td>
<td></td>
<td>2</td>
<td>1.5</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>Input $Q$-factor</td>
<td></td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>Inverted $Q$-factor</td>
<td></td>
<td>50.51 ± 2.15</td>
<td>48.92 ± 2.86</td>
<td>51.63 ± 3.52</td>
<td>53.46 ± 6.08</td>
</tr>
</tbody>
</table>

**Table 6. Tests for the tolerance to random noise.**

<table>
<thead>
<tr>
<th>Source wavelet</th>
<th>Reflectivity sequence</th>
<th>Trace 1</th>
<th>Trace 2</th>
<th>Trace 3</th>
<th>Trace 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak frequency (Hz)</td>
<td>Sparsity coefficient</td>
<td>60</td>
<td>0.6</td>
<td>0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>Traveltime (s)</td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Gaussian noise (%)</td>
<td></td>
<td>0</td>
<td>1</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>Input $Q$-factor</td>
<td></td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>Inverted $Q$-factor</td>
<td></td>
<td>51.60 ± 3.16</td>
<td>48.69 ± 3.57</td>
<td>49.10 ± 4.24</td>
<td>52.44 ± 5.21</td>
</tr>
</tbody>
</table>

**CASE STUDY**

Figure 16 is a poststack seismic profile in the Pearl River Mouth Basin, northern South China Sea. In this area, gas chimneys are widely developed (Sun, Q. et al., 2012, Sun, Y. et al., 2012). As shown in the profile, the black solid line delineates the location of a gas chimney. It is easily identified by its unique seismic characteristics of disrupted reflections, dim amplitude anomalies, low-continuity, and enhanced reflections above the gas chimney (Sun, Y. et al., 2012). As we know, gas within the sediments causes an apparent decrease in the $Q$-factor. Thus, we use these data to verify the FWE inversion method for the interval $Q$-factor inversion.

Because a single stacked reflection event is composed of signals from multiple offsets, each representing a different travel path, path length, angle of incidence, and source-receiver pair (Hackert and Parra, 2004), it is not accurate to invert the $Q$-factor from poststack seismic reflection data. To ensure the accuracy, we can use the full...
prestack gathers (Dasgupta and Clark, 1998; Behura and Tsvankin, 2009; Reine et al., 2012a) or we can directly use the prestack zero-offset data.

Figure 17 is a profile composed of minimum-offset traces extracted from the shot gathers. Because the frequency characteristics of the seismic signal are crucial to inverting the $Q$-factor, the processing steps that possibly change the spectra of the data, such as deconvolution and frequency compensation, have not been performed. We only do some necessary denoising, e.g., swell noise, random noise at low and high frequencies, and linear noise. All these noises can be clearly separated from the effective signals in the frequency or $f-k$ domains, so that the effective signals can be fully preserved. Two horizons (H1 and H2 in Figure 17) are picked in the profile. The gas chimney is included in the sediments between these two horizons. Next, we invert the interval $Q$-factors of the sediments by the FWE inversion method, as shown in Figure 18a.

In Figure 18a, the $Q$-factors present an apparent low value between traces 330 and 360 (gray-shaded zone), whereas in the other traces, they have a relatively stable value of approximately 100–120. Compared with the seismic profile in Figure 17, the traces in which there are low $Q$-factors, are in good agreement with the location of the gas chimney. This confirms the reliability of the inversion.

To do comparisons, we also use the FWE, LSR, CFS, and PFS methods to estimate the interval $Q$-factors of the sediments based on the spectral correction method by estimating the wavelets (Tu and Lu, 2010). Each trace between H1 and H2 in the profile is divided into two halves from the middle. We estimate wavelets based on higher order statistics for both halves and consider them as the source and attenuated wavelet, respectively. Then, the $Q$-factor can be estimated by using the FWE, LSR, CFS, and PFS methods.

Taking trace 350 for example, Figure 19 illustrates the source and attenuated wavelet extracted from the trace between H1 and H2. The ampli-
Amplitude spectra of the estimated wavelets are smooth, but there are still tiny fluctuations at high frequencies. Hence, we define a threshold of 10% of the maximum spectra value. In every trace, we use the spectrum value above the threshold to calculate the centroid frequencies and variances of the source and attenuated wavelet in the FWE and CFS methods. The modeled FWE and CFS spectra of the wavelet spectra are also shown in Figure 19. As we can see, the modeled FWE spectra match the wavelet spectra better than the modeled CFS spectra.

Figure 20 illustrates the LSR of the source and attenuated wavelet versus frequency at traces 300, 350, and 450. The effective LSR value is located in similar bandwidth. Hence, in the LSR method, we choose the bandwidth of 30–100 Hz to fit the slope in every trace.

Figure 18 shows the interval $Q$-factor estimates of the FEW, LSR, CFS, and PFS methods by using the estimated wavelets, respectively. Comparing Figure 18 with Figure 18, the abnormality of the gas chimney in Figure 18 is clearer than that in Figure 18. Although Figure 18 looks similar to Figure 18, the contrast of the $Q$-factor between the gas-bearing sediments and the gas-free sediments in Figure 18 is more apparent. These indicate the advantages of the FWE method. However, comparing Figure 18 with Figure 18, the $Q$-factor estimates by using the estimated wavelets show less stability than that by the inversion method. This further proves that using multiple spectra in one time interval to invert the $Q$-factor makes the results more stable than using only two spectral estimates.
DISCUSSION

Because the stability of Q-factor inversion is highly dependent on the attenuation intensity, as we analyzed previously, the FWE inversion method needs long-time input data. The intervals chosen for this approach should be identified by zones of similar properties. By using the FWE inversion approach, we can get the interval Q-value that can be used for hydrocarbon detection and reservoir characterization of the target sediments. For the sediments in which the Q-factor has large variations, the interval Q-factor inverted by the approach can be treated as the average Q-value of the target sediments. Reducing the size of the interval measurements helps resolve the problem of a changing Q, but caution must be exercised because this also increases the uncertainty in the measurement.

For an entire seismic trace, we can divide it into several large sets of sediments according to the geologic or reflection characteristics. For each set of sediments, we invert an interval Q-factor. By this way, we get the Q-factor versus depth. We can use it to do compensation and Q-PSDM in the processing.

CONCLUSIONS

When a source wavelet propagates in attenuating media, we can use the FWE function to fit the amplitude spectra of the source and attenuated wavelet to obtain the symmetry indexes and bandwidth factors. After recalculating the bandwidth factors based on the average of the symmetry indexes, the Q-factor can be estimated by the decrease in the bandwidth factor between the source and the attenuated wavelet. Compared with the existing commonly used methods, our method applies to various source wavelet types and has better tolerance to random noise.

For seismic reflection data, we can use the FWE function to fit the local spectral contents of the seismic reflection data to obtain the symmetry indexes and bandwidth factors. After recalculating the bandwidth factors with the average of the symmetry indexes, we linearly fit the reciprocal of the bandwidth factor versus traveltime, and then the Q-factor can be calculated by the slope. This approach has high accuracy without having to correct for the tuning effect of data when the attenuation is high. It is not restricted by the dominant frequency of the source wavelet and the sparsity coefficient of the reflectivity sequence, and it also has good tolerance to random noise. Application of our approach in the field-data case study produced good results.

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