A new method to calculate the productivity of fractured horizontal gas wells considering non-Darcy flow in the fractures

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A B S T R A C T

A multiply fractured horizontal well is one of the most effective methods for improving the productivity of low permeability reservoirs. There could be large errors when calculating the productivity of fractured horizontal gas wells if inertial non-Darcy flow in the fractures is not considered. However, there are few reports about the deliverability equation of fractured horizontal wells influenced by non-Darcy effects due to the difficulty in coupling non-Darcy flow in the fractures and radial Darcy flow in the reservoir. A new concept named apparent equivalent wellbore radius, which considers inertial non-Darcy flow in a fracture, is proposed. An iterative process is presented to calculate the productivity of fractured vertical gas wells using the new concept. Then, the iterative process is extended to calculate the productivity of multiply fractured horizontal gas wells using the advantage of the apparent equivalent wellbore radius, which conforms to the superposition principle. The results show that inertial non-Darcy effects could decrease well production greatly even in low permeability reservoirs (<1 mD) and reduce potential interference between hydraulic fractures.

1. Introduction


Then, great achievements were made in the theories of computing the productivity of MFHWs and analyzing the pressure transient of MFHWs. Soliman et al. (1990), Larsen and Hegre (1994), Wan and Aziz (2002), and Wang et al. (2014a) presented type curves for MFHWs with constant production rates. In terms of well productivity, Raghavan and Joshi (1993) used an equivalent wellbore radius and the superposition principle to calculate the productivity of MFHWs with infinite conductivity fractures. Li et al. (1996) used a reformed flow resistance theory to predict the performance of a horizontal well that was hydraulically fractured and partially perforated. Wang and Li (2010) applied Raghavan’s idea to the case of MFHWs with finite conductivity fractures.

The flow theory of real gas is more difficult than that of oil due to non-Darcy effects caused by high-velocity gas, making studies on fractured vertical wells insufficient. Thus, based on over 2000 high-accuracy numerical solutions, Settari et al. (2002) developed a new correlation for vertically fractured wells and demonstrated the importance of turbulence for the productivity of oil wells, particularly in horizontal wells with transverse fractures. Smith et al. (2004) investigated the detrimental effects of non-Darcy flow on hydraulically fractured oil and gas wells and concluded that non-Darcy flow could lead to a reduction ranging from 10% to 35% in well performance. Miskimins and Lopez-Hernandez (2005) built a spreadsheet model based on the concept of pseudo-Reynolds number to estimate the effects of non-Darcy flow on production. The results demonstrated that non-Darcy flow could influence well...
production across the entire spectrum of flow rates.

Lopez-Hernandez et al. (2004) proposed a new concept named effective proppant number to select the appropriate β factor correlation. Also based on the concept of apparent fracture permeability, he optimized fracture treatment design to minimize the impact of non-Darcy flow through an iterative process. Rahman (2008) incorporated non-Darcy pressure loss into the mathematical model of equivalent wellbore radius for finite conductivity fractures. Then, a hybridized production model combining a transient flow regime with pseudo-steady state (PSS) flow regime was presented to predict the productivity of fractured vertical wells. According to a new concept named conductive thickness between equivalent radii (CTER), Wang et al. (2014b) developed a novel binomial deliverability equation for fractured gas wells considering non-Darcy effects in the fractures. The authors proposed a new concept named conductive thickness between equivalent radii (CTER), Wang et al. (2014b) developed a novel binomial deliverability equation for fractured gas wells considering non-Darcy effects. The equation conformed to traditional deliverability test theories of a vertical gas well.

The objective of this paper is to present a novel method to calculate the productivity of MFHWs considering non-Darcy effects in the fractures. The authors proposed a new concept named apparent equivalent wellbore radius and deduced a new deliverability equation for MFHWs. Then, an iterative process was presented, which could (1) calculate the production of multiply fractured horizontal gas wells considering non-Darcy effects in the fractures and (2) evaluate the effects of non-Darcy flow on well production. The novel method is useful to predict the production of a fractured horizontal gas well with the help of material balance theories when the well reaches the pseudo-steady state.

2. Basic theories

2.1. Equivalent wellbore radius model for infinite conductivity fractures

Based on the theories of Prats et al. (1962) and Gringarten and Ramey (1974), the equivalent wellbore radius for infinite conductivity fractures can be expressed as:

\[ r_{wef} = 0.499 x_f \]  

(1)

2.2. Dimensionless fracture conductivity

Dimensionless fracture conductivity has a great physical significance on hydraulic fractures and has been widely used in well productivity evaluation and transient well tests:

\[ C_{fD} = \frac{k_f \cdot w_f}{k_r \cdot x_f} \]  

(2)

2.3. Equivalent wellbore radius model for finite conductivity fractures

The equivalent wellbore radius model for infinite conductivity fractures may enlarge gas production. Thus, Wang and Li (2010) deduced an equivalent wellbore radius model for finite conductivity fractures:

\[ r_{wef} = 2x_f \exp \left\{ - \frac{3}{2} + f(C_{fD}) + S \right\} \]  

(3)

where

\[ f(C_{fD}) = \sum_{n=1}^{\infty} \frac{\pi C_{fD}}{2n + \pi C_{fD}(n + 1)} - \frac{\pi C_{fD}}{\pi C_{fD} + 2} \]  

(4)

It is difficult to calculate Eq. (4). Thus, an effective regression equation for Eq. (4) was presented by Wang et al. (2012):

\[ f(C_{fD}) = 0.95 - 0.56\phi + 0.16\phi^2 - 0.028\phi^3 + 0.0028\phi^4 - 0.00011\phi^5 \]  

\[ + 0.094\phi^6 + 0.093\phi^7 + 0.0084\phi^8 + 0.001\phi^9 + 0.00036\phi^{10} \]  

(5)

where, \( \phi = \ln(C_{fD}) \).

Rahman (2008) presented another equation to calculate the equivalent wellbore radius for finite conductivity fractures, which was expressed as:

\[ r_{wef} = \frac{r_w}{e^{-s}} \]  

(6)

Considering a bilinear flow characteristic in a finite conductivity fracture, Cinco-Ley and Samaniego (1981) proposed a graphical method for determining the pseudo-skin \( s_f \) in Eq. (6). The pseudo-skin can be obtained from the following relationship (Rahman, 2008):

\[ F = s_f + \ln \frac{x_f}{r_w} \]  

(7)

Eq. (7) could be changed into:

\[ s_f = F - \ln \frac{x_f}{r_w} \]  

(8)

Substituting Eq. (8) into Eq. (6), we derive a more concise expression for the equivalent wellbore radius for finite conductivity fractures without the influence of wellbore radius:

![Fig. 1. Equivalent effects comparison between equivalent wellbore radii.](image-url)
A simple but accurate programmable equation for calculating $F$ was presented by Rahman (2008):

$$F = \frac{1.65 - 0.328\sigma + 0.116\sigma^2}{1 + 0.18\sigma + 0.064\sigma^2 + 0.005\sigma^3}$$  \hspace{1cm} (10)$$

$$\sigma = \ln\left(\frac{C_{\Delta}}{C_{ID}}\right)$$  \hspace{1cm} (11)$$

Fig. 1 compares the results of Eq. (3) and Eq. (9). It shows that the equivalent wellbore radius values calculated by Eq. (9) are higher than those calculated by Eq. (3). When $C_{\Delta} > 100$, the equivalent wellbore radius is 0.493 when calculated by Eq. (9) and 0.443 by Eq. (3). Because the value of 0.493 is closer to the result of infinite conductivity fracture, Eq. (9) was chosen in the following parts.

3. Apparent equivalent wellbore radius

Apparent fracture permeability is an effective method for simulating inertial non-Darcy flow in the fracture. Miskimins and Lopez-Hernandez (2005) presented the derivation of apparent fracture permeability in detail. Rahman (2008) then used it to predict the production of fractured gas wells. The apparent fracture permeability was written as:

$$k_{f\_app} = \frac{k_f}{1 + N_{Re}}$$  \hspace{1cm} (12)$$

Substituting Eq. (12) into Eq. (2) yields:

$$C_{\Delta\_app} = \frac{k_{f\_app} \cdot \gamma_{g} \cdot w_f}{k_f \cdot \gamma_{g} \cdot \left(1 + N_{Re}\right) \cdot \frac{\gamma_{g} \cdot \left(1 + \delta_{f}\right) \cdot \gamma_{g} \cdot w_f}{k_f \cdot \gamma_{g} \cdot \left(1 + N_{Re}\right)}}$$  \hspace{1cm} (13)$$

Thus, a new concept of apparent dimensionless fracture conductivity is proposed:

$$C_{\Delta\_app} = \frac{C_{\Delta}}{1 + N_{Re}}$$  \hspace{1cm} (14)$$

The physical meaning of apparent dimensionless fracture conductivity is as follows: apparent dimensionless fracture conductivity considers the effects of non-Darcy flow and is a function of dimensionless fracture conductivity and the Reynolds number.

Finally, by virtue of the apparent dimensionless fracture conductivity, another new concept named apparent equivalent wellbore radius is put forward, which is expressed mathematically as follows:

$$r_{wef\_app} = x_f e^{-F_{app}}$$  \hspace{1cm} (15)$$

in which:

$$F_{app} = \frac{1.65 - 0.328\delta + 0.116\delta^2}{1 + 0.18\delta + 0.064\delta^2 + 0.005\delta^3}$$  \hspace{1cm} (16)$$

$$\delta = \ln\left(\frac{C_{\Delta\_app}}{C_{ID}}\right)$$  \hspace{1cm} (17)$$

The physical definition of apparent equivalent wellbore radius is as follows: the equivalent wellbore radius for a hydraulic fracture considers fracture conductivity and inertial non-Darcy effects in the fracture simultaneously.

The mathematical expression of the Reynolds number in Eq. (12) is deduced below.

The Reynolds number used by Lopez-Hernandez et al. (2004) was:

$$N_{Re} = 1.83 \times 10^{-16} \frac{b_k \rho_{g} \gamma_{g}}{\mu_{g}}$$  \hspace{1cm} (18)$$

where

$$\rho_{g} = \frac{P_{wef} M_{g}}{2R T}$$  \hspace{1cm} (19)$$

$$M_{g} = M_{\text{air}} \cdot \gamma_{g}$$  \hspace{1cm} (20)$$

Gas flow velocity is (Rahman, 2008):

$$v_{g} = \frac{500B_{g} q_{sc}}{h_f w_f}$$  \hspace{1cm} (21)$$

where

$$B_{g} = 0.0282 \frac{zT}{P_{wef}}$$  \hspace{1cm} (22)$$

Substituting Eqs. (19)–(22) into Eq. (18), we get:

$$N_{Re} = 25.803 \times 10^{-16} \frac{b_k M_{\text{air}} \gamma_{g} q_{sc}}{\mu_{g} h_f w_f}$$  \hspace{1cm} (23)$$

In this paper, $R$ is constant, i.e., 10.732; $M_{\text{air}}$ is the molecular weight of gas, 29; and $\gamma_{g}$ is the gas specific gravity, 0.644.

Fig. 2 also shows influences of Reynolds number on equivalent effects of the apparent equivalent wellbore radius. The Reynolds numbers in Fig. 2 are in the range from 0 to 16. If $C_{\Delta}$ is less than 1000, the apparent equivalent wellbore radius will decrease with increasing Reynolds number at a certain $C_{\Delta}$. Meanwhile, if $C_{\Delta}$ is greater than 1000, the fracture will be close to infinite conductivity and insensitive to the Reynolds number. The result infers that improving dimensionless fracture conductivity is an effective way to improve the apparent equivalent wellbore radius.

3.1. Model validation

The model of the apparent equivalent wellbore radius was validated by comparing our model with the model of Rahman (2008) in Fig. 2. When the Reynolds number is zero, the result of our model coincides with that of Rahman.

The following is a list of the fundamental properties of the gas, reservoir, and fractures used in this study.

<table>
<thead>
<tr>
<th>Name of parameters</th>
<th>Basic data</th>
<th>Name of parameters</th>
<th>Basic data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir height, $h$</td>
<td>20 ft</td>
<td>Fracture permeability, $k_f$</td>
<td>10000 mD</td>
</tr>
<tr>
<td>Bounded pressure, $P_b$</td>
<td>4400 psi</td>
<td>Compressibility factor, $Z$</td>
<td>0.89</td>
</tr>
<tr>
<td>Bottom hole pressure, $P_{wef}$</td>
<td>1600 psi</td>
<td>Reservoir radius, $R$</td>
<td>2500 ft</td>
</tr>
<tr>
<td>Reservoir temperature, $T$</td>
<td>660 $^\circ$R</td>
<td>Fracture half length, $x_f$</td>
<td>200 ft</td>
</tr>
<tr>
<td>Gas viscosity, $\mu_{g}$</td>
<td>0.027 cp</td>
<td>Fracture width, $w$</td>
<td>0.01 ft</td>
</tr>
<tr>
<td>Reservoir permeability, $k_r$</td>
<td>0.2 mD</td>
<td>Inertial coefficient, $\beta$</td>
<td>41.462 1/ft</td>
</tr>
<tr>
<td>Fracture Height, $h_f$</td>
<td>20 ft</td>
<td>Fracture distances, $d$</td>
<td>400 ft</td>
</tr>
</tbody>
</table>
3.2 Analyses and discussion

Fig. 3 shows the effects of well production and the inertial factor on apparent fracture permeability. As shown in Fig. 3, apparent fracture permeability decreases rapidly with increasing gas well production. Furthermore, the larger the inertial factor is, the more dramatically it will decrease.

Fig. 4 presents the effects of well production and the inertial factor on apparent equivalent wellbore radius. It shows that the apparent equivalent wellbore radius decreases with increasing well production and inertial factor (Table 2). For example, when \( q_{sc} = 1000 \text{ Mscfd} \) and \( \beta \) increases from 0 to 11462, 21462, 31462 and 41462 \( \text{ft}^{-1} \), the apparent equivalent wellbore radius will decrease by 5.4%, 9.7%, 13.7% and 17.3%, respectively. When \( q_{sc} = 4000 \text{ Mscfd} \) and \( \beta \) increases from 0 to 11462, 21462, 31462 and 41462 \( \text{ft}^{-1} \), the apparent equivalent wellbore radius will decrease by 18.8%, 30.4%, 39% and 45.7%, respectively.

Fig. 5 shows the influence of fracture width on apparent equivalent wellbore radius. It reflects that a wider fracture width could effectively improve the apparent equivalent wellbore radius to lower inertial non-Darcy effects.

Fig. 6 shows the influence of fracture permeability on apparent equivalent wellbore radius. The five curves coincide with each other closely when fracture permeability increases from 7000 mD to 110000 mD. This means that fracture permeability is a subordinate factor for minimizing inertial non-Darcy effects compared with fracture width.

4. Productivity model of a fractured gas well

4.1 Physical model and assumptions

(1) A homogeneous, isotropic, horizontal, circular slap reservoir is bounded by impermeable upper and lower stratum. The radius of the circular reservoir is \( R_e \).

(2) Gas is produced through a vertically fractured well intersected by a fully penetrating, finite conductivity fracture of half-length \( x_f \), width \( w_f \) and permeability \( k_f \). Inertial non-Darcy flow in the fracture is considered.

Table 2

<table>
<thead>
<tr>
<th>( q_{sc} ) (Mscfd)</th>
<th>( rw_{ef_app} ) (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>94.9</td>
</tr>
<tr>
<td>4000</td>
<td>94.9</td>
</tr>
<tr>
<td>( \beta = 0 )</td>
<td>89.8</td>
</tr>
<tr>
<td>( \beta = 11462 )</td>
<td>85.7</td>
</tr>
<tr>
<td>( \beta = 21462 )</td>
<td>81.9</td>
</tr>
<tr>
<td>( \beta = 31462 )</td>
<td>78.4</td>
</tr>
<tr>
<td>( \beta = 41462 )</td>
<td>75.1</td>
</tr>
</tbody>
</table>

4.2. The productivity model

The productivity equation of a fractured gas well is constructed by using the apparent equivalent wellbore radius and has the same form as the productivity equation of a vertical well. It could be written as:

\[ P_2 - P_{wf} = \frac{1422 T \mu g Z q_{sc}}{k_f h} \left( \ln \frac{R_e}{r_{wef,app}} \right) \]

(24)

According to Eq. (24), the production of a fractured gas well could be calculated by:

\[ q_{sc} = \frac{k_f h \left( P_2 - P_{wf} \right)}{1422 T \mu g Z} \left( \ln \frac{R_e}{r_{wef,app}} \right)^{-1} \]

(25)

\( r_{wef,app} \) is a function of \( q_{sc} \), and so the difficulty in calculating Eq. (25) is that \( q_{sc} \) is a function of \( q_{sc} \) itself. Hence, an iterative process, listed below, is needed to solve the equation:

1. Guess an initial Reynolds number \( (N_{Re0} = 9.8 \) in this paper) and then calculate \( k_f,app \) according to Eq. (12).
2. Calculate apparent dimensionless fracture conductivity according to Eq. (13).
3. Calculate apparent equivalent wellbore radius \( r_{wef,app} \) according to Eq. (15).
4. Calculate well production \( q_{sc} \) according to Eq. (25).
5. Calculate the new Reynolds number \( N_{Re1} \) according to Eq. (23).
6. If \( \frac{N_{Re1} - N_{Re0}}{N_{Re0}} < 0.01 \), then the production is \( q_{sc} \); otherwise, \( N_{Re0} = N_{Re1} \), the Reynolds number calculated in step (5) is used in step (1) and the calculations are repeated until the error in step (6) is <0.01%.

4.3. Model validation

Comparisons between our model and other models are shown in Fig. 7. When the Reynolds number is zero, the well production calculated by our model is less than that of Gringarten and Ramey, 1974 and is consistent with Wang and Li (2010). Our model can consider inertial non-Darcy effects in the fracture, while the others could not.

4.4. Analyses and discussion

Fig. 7 also depicts IPR curves (relationship between bottom hole flowing pressure (BHFP) and well production) for a fractured vertical gas well with varying inertial factors. Fig. 8 reflects the
relationship between BHFP and the Reynolds number. Fig. 9 shows the relationship between BHFP and apparent equivalent wellbore radius, indicating that when BHFP decreases, the well production will increase (Fig. 7). As a result, the Reynolds number increases (Fig. 8) and apparent equivalent wellbore radius decreases (Fig. 9). Furthermore, the higher the inertial non-Darcy factor is, the more drastic these changes will be. For example, when BHFP is 1000 psi and $\beta$ increases from 0 to 21462 ft, the Reynolds number will increase from 0 to 1.56, apparent equivalent wellbore radius will decrease from 94.9 ft to 86.73 ft (i.e., by 8.6%) and well production will decrease from 900.8 Mscfd to 876.6 Mscfd (i.e., by 2.7%). When BHFP is 1000 psi and $\beta$ increases from 0 to 41462 ft, the Reynolds number will increase from 0 to 2.96, apparent equivalent wellbore radius will decrease from 94.9 ft to 80.45 ft (i.e., by 15.2%) and well production will decrease from 900.8 Mscfd to 857.5 Mscfd (i.e., by 4.8%) at the same time.

Fig. 10 shows the effects of reservoir permeability on IPR curves. It is found that high reservoir permeability is in favor of the gas flow and could lead to high well production. However, inertial non-Darcy effects influence the high permeability reservoir more easily. For example, if $k_r$ is 0.2 mD and $\beta$ increases from 0 to 21462 ft and 41462 ft, $q_{sc}$ will decrease by 2.7% and 4.8%, respectively, and $r_{w,eq,app}$ will decrease by 8.6% and 15.22%, respectively. If $k_r$ is 0.5 mD and $\beta$ increases from 0 to 21462 ft and 41462 ft, $q_{sc}$ will decrease by 10.6% and 15.9%, respectively, and $r_{w,eq,app}$ will decrease by 32.9% and 47.2%, respectively.

Fig. 11 further analyzes the effects of reservoir permeability on apparent equivalent wellbore radius. When $\beta = 41462$ ft, $P_{w,f} = 1600$ psi, and reservoir permeability varies from 0.2 to 0.4, 0.6, 0.8, and 1 mD, $r_{w,eq,app}$ will decrease by 31%, 52.1%, 65.4%, and 74%, respectively. The results demonstrate that non-Darcy effects should not be neglected even in tight gas reservoirs whose permeability is less than 1 mD. Because of the adverse influence of non-Darcy effects on apparent equivalent wellbore radius and gas well production, it is necessary to optimize the fracturing treatment before hydraulic fracturing and well opening.

Fig. 12 shows how fracture half-length influences the IPR curves of a fractured well. The result shows that improving fracture half-length could increase well production and intensify non-Darcy effects at the same time. For example, when the reservoir permeability is 0.2 mD, fracture half-length is 200 ft, and $\beta$ increases from 0 to 41462 ft, $q_{sc}$ will decrease by 4.8%. When the reservoir permeability is 0.2 mD, fracture half-length is 300 ft, and $\beta$ increases from 0 to 41462 ft, $q_{sc}$ will decrease by 8%.

5. Productivity model of a fractured horizontal gas well

5.1. Physical model and assumptions

(1) The conditions of the reservoir are the same as the model of the fractured vertical gas well.

(2) Gas is produced through the horizontal well intersected by a random number of transverse fractures with the same properties. The horizontal well is in the center of the circular reservoir. The distance between two fractures $d$ is larger than the fracture half-length $x_f$. The length of the horizontal wellbore is less than the radius of the reservoir.

(3) The horizontal well has infinite conductivity. Gas flows from the reservoir to fractures and then from fractures to the horizontal wellbore. Inertial non-Darcy effects in fractures are considered. The schematic diagram of the fractured horizontal well with seven fractures is shown in Fig. 13.

5.2. The productivity model

Based on the theories of equivalent wellbore radius and the superposition principle, Raghavan and Joshi (1993) derived the productivity equation for a fractured horizontal oil well with three and five infinite conductivity fractures. Wang and Li (2010) applied
the method to a horizontal well with finite conductivity fractures and a random fracture number.

Eq. (24) has the same form as the productivity equation of a vertical well. Thus, the full-blown theories of a vertical well, e.g., the superposition principle, could be directly applied to a fractured horizontal gas well considering non-Darcy effects.

The general form of pressure square at a random observation point is obtained by applying the pressure-drop superposition principle (Wang and Li, 2010):

\[
p^2 = \frac{1422T\mu g Z}{k_i h} \left( \sum_{i=1}^{n} q_{sci} \ln r_i \right) + C
\]

where \( n \) is the total fracture number; \( P \) is the pressure of a random observation point, psi; \( q_{sci} \) is the production of fracture \( i \), Mscfd; \( r_i \) is the distance between the observation point and middle point of fracture \( i \), ft; and \( C \) is the pressure square of the reference point.

By virtue of apparent equivalent wellbore radius, the productivity of a fractured horizontal well considering non-Darcy effects can be calculated directly by using the superposition principle. If the observation points are located in the middle point of each fracture, the pressure square of fracture \( j \) could be expressed as:

\[
p^2_{wf(j)} = \frac{1422T\mu g Z}{k_i h} \left( \sum_{i=1}^{n} q_{sci} \ln r_{ij} + q_{sci} \ln r_{wef(app(j))} \right) + C
\]

where \( r_{ij} \) is the distance between fracture \( i \) and fracture \( j \), ft. If the distance between fractures is equal to \( d \), then \( r_{ij} = \text{Abs}(i-j)d \), where \( \text{Abs}(\cdot) \) is the absolute value sign.

Because \( r_{ij} < \text{Re}_{0} \), we approximate the distance between any fracture and reservoir boundary to be equal to the radius of a circular reservoir. As a result, the pressure square of a boundary can be expressed as:

\[
p^2_e = \frac{1422T\mu g Z}{k_i h} \left( \sum_{i=1}^{n} q_{sci} \ln r_e + q_{sci} \ln r_e \right) + C
\]

Using Eq. (28) minus \( n \) equations in Eq. (27), we deduce a new system of linear equations for the productivity of a fractured horizontal gas well:

\[
\begin{bmatrix}
\ln \frac{Re}{r_{wef(app(1))}} \\
\ln \frac{Re}{r_{wef(app(2))}} \\
\ldots \\
\ln \frac{Re}{r_{wef(app(n))}}
\end{bmatrix}
\begin{bmatrix}
q_{sci}1 \\
q_{sci}2 \\
\ldots \\
q_{sci}n
\end{bmatrix}
\times
\begin{bmatrix}
\frac{P^2_{e}-P^2_{wf(1)}}{P^2_{e}-P^2_{wf(n)}} \\
\frac{P^2_{e}-P^2_{wf(2)}}{P^2_{e}-P^2_{wf(n)}} \\
\ldots \\
\frac{P^2_{e}-P^2_{wf(n)}}{P^2_{e}-P^2_{wf(n)}}
\end{bmatrix}

\]

When BHFP is gauged, the production of each fracture can be calculated using Eq. (29). At last, the total production of the fractured horizontal well is the sum of the production from all fractures:

\[
Q_{sc} = \sum_{i=1}^{n} q_{sci}
\]

The difficulty in calculating Eq. (29) is that determinant \( q_{sci} \) is a function of \( q_{sci} \) itself. Hence, an iterative procedure extended from Eq. (25) is presented as follows:

1. Guess an initial Reynolds number for all fractures \( (N_{Re0(j)} \text{ is 9.8 in this paper}) \) and then calculate \( k_{f(app(j))} \) according to Eq. (12) and calculate \( C_{f(app(j))} \) according to Eq. (13) for each fracture.
2. Calculate apparent equivalent wellbore radius \( r_{wef(app(j))} \) for each fracture according to Eq. (15).
3. Calculate production \( q_{sci(j)} \) for all fractures according to Eq. (29).
4. Calculate the new Reynolds numbers \( N_{Re1(j)} \) according to Eq. (23).
5. If \( \frac{N_{Re1(j)}-N_{Re0(j)}}{N_{Re0(j)}} < 0.01\% \) is correct for all fractures, then the production in each fracture is \( q_{sci(j)} \) and the total production of the horizontal well is \( Q_{sc} = \sum_{i=1}^{n} q_{sci(j)} \); otherwise,

Fig. 13. Schematic diagram of the fractured horizontal well with seven fractures.
The Reynolds numbers calculated in step (4) are used in step (1) and the calculations are repeated until the errors in step (5) are < 0.01%.

5.3. Model validation

Comparisons between our model (five fractures) and other models are shown in Fig. 14. When the Reynolds number is zero, the well production calculated by our model is less than that of Raghavan and Joshi (1993) and is consistent with that of Wang and Li (2010). Our model can consider inertial non-Darcy effects in fractures, while the others could not.

5.4. Analyses and discussion

The effects of the inertial factor on the production distribution of different fractures are shown in Fig. 15 and Fig. 16. These two figures show that the production of a fractured horizontal well with seven fractures has symmetrical distribution and mainly comes from the outermost fractures, whose series numbers are 1 and 7. When the inertial factor increases, the total production will decrease. In this process, production of the two outermost fractures decreases dramatically, while production of the inner fractures increases mildly. For example, when reservoir permeability is 0.5 mD and $\beta$ increases from 0 to 41462 1/ft, the production of fracture 1 and fracture 7 decreases by 14%, production of fracture 2 and fracture 6 decreases by 2%, and production of fractures 3, 4, 5 decreases by 1%.

Table 3 Parameter distributions of a fractured gas well with seven fractures (varying inertial factor, $kr = 0.2$ mD).

<table>
<thead>
<tr>
<th>SN</th>
<th>$\beta = 0$</th>
<th>$\beta = 41462$</th>
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</thead>
<tbody>
<tr>
<td>qsc</td>
<td>Nre</td>
<td>$k_{r,app}$</td>
</tr>
<tr>
<td>1</td>
<td>578.1</td>
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</tr>
<tr>
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<td>252.4</td>
<td>0</td>
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<tr>
<td>3</td>
<td>224.4</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
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<td>224.4</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>252.4</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>578.1</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4 Parameter distributions of a fractured gas well with seven fractures (varying inertial factor, $kr = 0.5$ mD).

<table>
<thead>
<tr>
<th>SN</th>
<th>$\beta = 0$</th>
<th>$\beta = 41462$</th>
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</thead>
<tbody>
<tr>
<td>qsc</td>
<td>Nre</td>
<td>$k_{r,app}$</td>
</tr>
<tr>
<td>1</td>
<td>1403.7</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>640.0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>560.0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>534.6</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>560.0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>640.0</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>1403.7</td>
<td>0</td>
</tr>
</tbody>
</table>
increases by 7.2%, production of fracture 3 and fracture 5 increases by 1.7%, production of fracture 4 increases by 1.3%, and the total production of the fractures decreases by 4.8%. The specific data are presented in Table 3 and Table 4.

The effect of inertial factors on the fracture Reynolds number, apparent fracture permeability and apparent equivalent wellbore radius are shown in Figs. 17–19, respectively. When the inertial factor increases, the Reynolds number of each fracture increases, which leads to the decrease in the apparent fracture permeability and apparent equivalent wellbore radius of each fracture.

The analyses above indicate an interesting phenomenon. When the inertial factor increases, production of the two outermost fractures will decrease with increasing Reynolds number and decreasing apparent equivalent wellbore radius, while production of the inner fractures will increase with increasing Reynolds number and decreasing apparent equivalent wellbore radius. This indicates that a higher inertial factor could lower the potential interference. Because the production increase in the inner fractures is less than the production decrease in the two outermost fractures,
the total production of MFHW ultimately decreases.

The results presented thus far only concern the effects of a varying inertial factor on a fractured horizontal well. Fig. 20 investigates the effects of fracture half-length on the production distribution of each fracture. It shows that a longer fracture half-length could improve production of the two outermost fractures greatly and improve the production of fractures 3, 4, and 5 slightly. However, it could decrease the production of fracture 2 and fracture 6. As a result, with the increase in fracture half-length, production of the inner fractures would be closer to each other and approach a constant. Fig. 21 shows the effects of fracture half-length on the Reynolds number distribution of each fracture. The change in the Reynolds number of each fracture follows the same law as the change in production. It is concluded that a longer fracture half-length can greatly improve the potential interference between fractures.

Fig. 22 shows the effects of fracture half-length on the apparent equivalent wellbore radius of each fracture. It shows that a longer fracture half-length could improve the apparent equivalent wellbore radius of all fractures because they are proportional to the fracture half-length. Moreover, the radii of inner fractures are bigger than those of outer fractures.

Horizontal well models with five fractures and seven fractures were investigated. Our model can be applied to a horizontal well traversed by an arbitrary number of fractures. Fig. 23 shows the relationship between horizontal well production and fracture numbers. The detailed data are presented in Table 5. It is concluded that more fractures will increase horizontal well production when the fracture distance is constant.

Note that the cases we presented above include horizontal wells with the same fracture properties and distance. In fact, our model could be applied to a horizontal well with various fracture properties, such as different fracture half-lengths, widths, permeability, and unequal fracture distances.

### 6. Conclusions

A novel concept named apparent equivalent wellbore radius is proposed in this paper. Based on the new concept, iterative processes are presented to calculate the productivity of fractured vertical gas wells and multiply fractured horizontal gas wells. The following conclusions are drawn from this study:

1) The Reynolds number and apparent equivalent wellbore radius are two key factors for evaluating the adverse effects of inertial non-Darcy flow in a fracture. The Reynolds number is directly proportional to the inertial non-Darcy factor. Adverse non-Darcy effects with a higher Reynolds number can greatly decrease apparent fracture permeability and apparent equivalent wellbore radius. Enhancing fracture width is an effective way of lowering the inertial non-Darcy effects. Moreover, improving fracture permeability can reduce adverse effects mildly.

2) Inertial non-Darcy effects have adverse effects on the productivity of fractured vertical and horizontal wells. It is found that higher reservoir permeability favors gas flow and can lead to higher well production. However, inertial non-Darcy effects can influence high permeability reservoirs more easily. Meanwhile, though improving fracture half-length intensifies non-Darcy effects, it will also improve well production. The results demonstrate that non-Darcy effects can not be neglected even in low permeability gas reservoirs ($k_r = 0.2$ mD). It is necessary to optimize the fracturing treatment before well opening.

3) For multiply fractured horizontal wells with a fixed fracture length, when the inertial factor increases, the total production decline will be mainly caused by the production decrease of the two outermost fractures. Though the Reynolds numbers of inner fractures increase and their equivalent wellbore radii decrease at the same time, the production of inner fractures increases. This indicates that a higher inertial factor causes higher adverse non-Darcy effects, which can lower the potential interference between multiple fractures.

### Acknowledgments

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### Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_g$</td>
<td>real gas viscosity, cp;</td>
</tr>
<tr>
<td>$\beta$</td>
<td>inertial coefficient, 1/ft;</td>
</tr>
<tr>
<td>$\rho$</td>
<td>gas density, lbm/ft$^3$;</td>
</tr>
<tr>
<td>$\gamma_g$</td>
<td>gas specific gravity;</td>
</tr>
<tr>
<td>$B_g$</td>
<td>gas volumetric factor, rcf/scf</td>
</tr>
<tr>
<td>$C_f$</td>
<td>fracture conductivity, mD-ft</td>
</tr>
<tr>
<td>$C_{D,app}$</td>
<td>apparent dimensionless fracture conductivity</td>
</tr>
<tr>
<td>$\delta$</td>
<td>identical distances between fractures, ft</td>
</tr>
<tr>
<td>$F$</td>
<td>function of dimensionless fracture conductivity</td>
</tr>
<tr>
<td>$F_{app}$</td>
<td>function of apparent dimensionless fracture conductivity</td>
</tr>
<tr>
<td>$FN$</td>
<td>total fracture numbers of fractured horizontal well;</td>
</tr>
<tr>
<td>$h$</td>
<td>reservoir height, which is equal to fracture height, ft;</td>
</tr>
<tr>
<td>$h_f$</td>
<td>fracture height, ft</td>
</tr>
<tr>
<td>$k_r$</td>
<td>reservoir permeability, mD</td>
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### Table 5

Production of MFHW with different fracture numbers.

<table>
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<th>FN</th>
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<td>$\beta = 0$</td>
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<td>2</td>
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<td>1147.5</td>
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<tr>
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<td>9</td>
<td>2934.5</td>
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<td>10</td>
<td>3237.7</td>
<td>3224.7</td>
</tr>
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</table>
\( k_f \)  
\( k_{f,app} \)  
\( M_g \)  
\( M_{air} \)  
\( N_{Re} \)  
\( P \)  
\( P_e \)  
\( P_{wf} \)  
\( P_{wf}(j) \)  
\( q_{sc} \)  
\( q_{sci} \)  
\( q_{sj} \)  
\( Q_{sc} \)  
\( R_e \)  
\( r_w \)  
\( r_{wef} \)  
\( r_{wef,app} \)  
\( r_{wef,app}(j) \)  
\( r_{ij} \)  
\( s_f \)  
\( SN \)  
\( T \)  
\( V_g \)  
\( w_f \)  
\( X_f \)  
\( Z \)  

References


