An extension of gravity probability tomography imaging

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1. Introduction

Gravity inversion is an important tool for retrieving the model parameters from measured gravity data. It is widely used for mapping geological structures in tectonic studies, resource exploration and engineering investigations, especially at the reconnaissance stage of these applications.

Inversion methods have undergone considerable development in the past decades. One type of gravity inversion is the direct determination of the three-dimensional subsurface distribution of density contrasts and includes both linear and nonlinear methods. Li and Oldenburg (1996) proposed two such techniques. In the first of these, the gravity data is transformed into pseudomagnetic data using the Poisson’s relation, and the inversion is carried out using a 3-D general magnetic inversion algorithm (Li and Oldenburg, 1996). In the second technique, the gravity data is inverted to recover a minimum structure model, and the final density distribution is obtained by minimizing a model objective function. Nonlinear inversion methods are also widely used in gravity inversion. Bosch et al. (2006) used Monte Carlo simulation techniques in gravity inversion. Chassereau and Chouteau (2003) advocated 3-D inversion of gravity data using an a priori model of covariance. Shamsipour et al. (2010a,b, 2011) proposed a geostatic inversion technique for the inversion of gravity data obtained at ground surface and in boreholes.

The second kind of gravity inversion is probability tomography, which deals with the subsurface distribution of contrasting densities in a purely probabilistic sense without external constraints. Probability tomography was first developed for the analysis of self-potential data (Patella, 1997a,b) and then extended to geoelectric and electromagnetic methods (Mauriello and Patella, 1999a,b; Mauriello et al., 1998). Gravity and magnetic probability tomography imaging have also been developed as potential methods (Chianese and Lapenna, 2007; Guo et al., 2011a,b; Mauriello and Patella, 2001, 2005, 2008).

In gravity or magnetic inversion, non-uniqueness of the solution poses a problem for the mathematical properties of potential fields, in that many subsurface density or magnetic distributions produce identical responses. Inversion methods always need to impose constraints to help guide the inversion and obtain robust results (Bosh and McGaughey, 2001; Boulanger and Chouteau, 2001; Fullagar et al., 2008).

In comparison with methods that directly invert density contrast, probability tomography of gravity data is simple, stable and readily performed. Since the results are probabilities and not actual density contrasts, they take values between −1 and +1; it is also difficult to add geological constraints in the image processing to overcome the inherent non-uniqueness and to improve resolution. In this paper, we propose an iterative method of density contrast inversion that incorporates probability tomography computation of the mismatch between the observed
gravity data and forward gravity data for a given density model. A straightforward constraint is also added to the inversion procedure that restricts the range of inverted density values to yield a geologically meaningful result.

### 2. Review of gravity tomography theory

In a reference Cartesian coordinate system (x-y) plane horizontal and z-axis positive downwards, the subsurface is divided into a large number of rectangular cells of constant density. Then, we assume that all gravity reading stations P(x,y,z) are located on the ground surface at varying elevations |z| above mean sea level. Referring to a cell q with differential density \( \Delta \rho \) with respect to the host material (Fig. 1a), the Bouguer anomaly \( \Delta g(x,y,z) \) is written as:

\[
\Delta g(x,y,z) = \frac{G \Delta \rho v_q (z_q - z)}{(x_q - x)^2 + (y_q - y)^2 + (z_q - z)^2}^{1/2}
\]

where \( G \) is the universal gravitational constant; \( v_q \) is the volume of a rectangular cell; and \( \Delta \rho \) can be positive or negative, depending on the presence of an anomalous excess or deficit mass in cell q with respect to the host volume. Summing all cells \( Q \) in the region (Fig. 1b), the total Bouguer gravity anomaly \( \Delta g(x,y,z) \) is given by:

\[
\Delta g(x,y,z) = G \sum_{q=1}^{Q} \Delta \rho v_q (z_q - z)
\]

where the subscripts \( i \) of x, y, z refer to a station point.

For a unit rectangular mass \( \Delta \rho v_q = 1 \), the Bouguer anomaly is expressed as:

\[
\Delta g(x,y,z) = G \sum_{i=1}^{N} \frac{(z_q - z)}{(x_q - x)^2 + (y_q - y)^2 + (z_q - z)^2}^{1/2}
\]

Based on the derivation by Mauriello and Patella (2001), the probability tomography imaging function \( \eta_q \) of a rectangular cell is obtained from:

\[
\eta_q = \left[ \frac{\sum_{i=1}^{N} \Delta g(x_i, y_i, z_i) \Delta g_0(x_i, y_i, z_i)}{\Delta g_0^2(x_i, y_i, z_i) \sum_{i=1}^{N} \Delta g_0(x_i, y_i, z_i)} \right]^{1/2}
\]

where \( N \) is the number of station points on the ground surface.

Substituting from Eqs. (3), (4) gives:

\[
\eta_q = \left[ \frac{\sum_{i=1}^{N} \Delta g(x_i, y_i, z_i) B_q(x_i, y_i, z_i)}{\sum_{i=1}^{N} \Delta g_0^2(x_i, y_i, z_i) \sum_{i=1}^{N} B_q^2(x_i, y_i, z_i)} \right]^{1/2}
\]

where:

\[
B_q(x_i, y_i, z_i) = \frac{(z_q - z)}{(x_q - x)^2 + (y_q - y)^2 + (z_q - z)^2}^{1/2}
\]

Fig. 1. (a) Rectangular cell q of density \( \rho_q \) within host volume of density \( \rho_i \). (b) Division of subsurface into rectangular cells with constant density value.

Fig. 2. (a) Geological model A consisting of a simple prism. (b) Probability imaging result for model A; the color scales represent probability value between -1 and +1.
takes the role of a space-domain scanning function in the probability tomography method. Using the 2-D Cauchy–Schwarz inequality property, we write:

\[
\left( \sum_{i=1}^{N} \Delta g_i(x_i, y_i, z_i) B_i(x_i, y_i, z_i) \right)^2 \leq \sum_{i=1}^{N} \Delta g_i^2(x_i, y_i, z_i) \sum_{i=1}^{N} B_i^2(x_i, y_i, z_i)
\]  

(7)

Then we note that the probability function satisfies the condition:

\[-1 \leq \eta \leq +1.\]  

(8)

For given observed gravity data on a 2-D grid, we subdivide a given subsurface region into a large number of rectangular cells. For all of these, we use Eq. (5) to calculate the probability function value \(\eta_i\) and then obtain a subsurface probability function imaging.

Positive values of \(\eta_i\) indicate the presence of excess density in the rectangular cell, while negative values are the result of a density deficit at the same point with respect to the host volume.

To specify the relationship between \(\eta_i\) and density contrast, a 2-D geological model was designed in the form of a simple prism buried in a zero-density background (Fig. 2a). The prism density was 0.5 g/cm\(^3\) located at 900–1000 m in \(x\) direction and 150–300 m at \(z\) direction. When forward image processing, the geology model was divided into 50 × 50 = 2500 cubic cells with dimension 50 × 50 m. Fig. 2b shows the probability tomography imaging result, which indicates the presence of an anomaly, but only within a broad range.

Multiplying a small density by the probability imaging result converts the probability value to physical parameter density. Forward modeling of this arrangement was compared to the forward gravity curve line in the original forward model (Fig. 3). If, when multiplying the probability value by a certain density value, the forward modeling curve has the same trend as the original data, and when we use the different density value, the difference between original and forward modeling curve will be different, but there is a density value that can minimize this difference. From this procedure, we have an indirect indication of the relationship between \(\eta_i\) and the density contrast proposed above.

3. Extension of gravity probability tomography

As discussed above, probability tomography imaging is a stable, fast and easy method, but its results do not give a direct indication of image density but rather a probability value between \(+1\) and \(-1\), which indicates the density influence and deficit with respect to the host volume. Multiplying a small constant density value converts the probability model into a density model, but it does not fit the observed data very closely. To remedy this, the present study proposes an iterative procedure for converting the probability value to a density value using an inversion method based on probability tomography.

Assuming that the host volume density is \(\rho\), a rectangular cell has a density of \(\rho + \Delta \rho\), after forward modeling and probability tomography imaging, the forward gravity data is \(\text{data}1\); the probability value for the rectangular cell is \(\eta\). If \(\eta\) is positive, \(\Delta \rho\) is also positive; if not, then \(\Delta \rho\) is negative. We multiply \(\eta\) by a density value and obtain a new density value \(\rho + \Delta \rho\) for the rectangle. We do the forward modeling with \(\rho + \Delta \rho\), giving forward gravity data \(\text{data}2\). Carrying out the probability tomography for \(\text{data}1–\text{data}2\) gives the new imaging value \(\eta_1\). This is the probability value of \(\Delta \rho\). Multiplying a density value \(\Delta \rho\) by \(\eta_1\) gives the perturbation of the inversion procedure; hence, \(\rho + \Delta \rho + \eta_1^\star \Delta \rho\) is the revised density model, \(\rho + \Delta \rho\) is the initial density model that provided by multiplying \(\eta\) with a density value.

Based on the above theory, the procedure for converting from probability value to physical density is summarized as follows:

1. Observe gravity data \(\text{data}1\), and perform probability tomography imaging \(\eta\).
2. Multiply \(\eta\) by a density value to convert the probability model to density model \(m1\). This is the first approximation of the final density model.
3. Forward-model \(m1\) to obtain gravity data \(\text{data}2\).
4. Carry out probability tomography imaging of \(\text{data}1–\text{data}2\) to obtain \(\eta_1\).
5. The new density model is then \(\rho + \eta_1^\star \Delta \rho\).
6. Repeat steps 3–5 until the average error value of \(\text{data}1\) and the new forward modeling satisfies the given threshold value.

Fig. 3. Original and forward gravity data curves. (a) Multiplying by a density value of 0.03. (b) Multiplying by a density value of 0.055.
In step (6), the average error is defined as:

$$\sigma_{av} = \frac{1}{N} \sum_{i=1}^{N} |\varepsilon_i|$$

Where $\varepsilon_i$ is the difference of initial data and forward modeling data of observation station $i$, $N$ is the station number on the surface. We usually give an initial $\sigma_{av}$ based on the forward modeling data of initial model and then change it based on the imaging result. For different data, the initial threshold of $\sigma_{av}$ to finish the inversion may be different because of the order of magnitude of original gravity data.

The above procedure was tested on two sets of synthetic model data. The first was the simple prism model described above; after probability imaging, the above inversion procedure was conducted as described. At the beginning of inversion, we give the threshold of average value is 0.08 and density contrast is 0.03 g/cm$^3$ that usually is the maximum density of research field divided by maximum value of probability imaging result; after three iterations, the inversion finished, and then we change the density contrast to 0.005 g/cm$^3$ and average value to 0.05 to finish the inversion; in the second time, we usually change the density contrast to a more little value to prevent the forward data changing in the vicinity of the original data. Fig. 4 shows the inversion result, where the imaging value represents density. Comparison with Fig. 2b shows an improved focusing effect.

The second model consists of two prisms, both of density 0.1 g/cm$^3$, buried at different depths in a zero-density background. The 3-D perspective model is shown in Fig. 5a. For forward and inversion processing, the model is divided into 20 $\times$ 20 $\times$ 20 = 8000 rectangular prisms with dimensions 50 $\times$ 50 $\times$ 25 m in the $x$, $y$, and $z$ directions respectively. The model produces the surface gravity anomaly shown in Fig. 5b. It consists of 400 pieces of data over a 20 $\times$ 20 grid of 50 m spacing. Fig. 6 is four slices of probability imaging result, the horizontal slices can indicate that there are two anomalies but they are failed to separated in vertical slices. Using this probability imaging result, we invert this...
dataset using this method provided in this paper. At the beginning of inversion, we give that the threshold of average value is 0.1 and density contrast is 0.093 g/cm³; after four iterations, the inversion finished, and then we change the density contrast to 0.005 g/cm³ and average value to 0.06 to finish the inversion.

Four inversion slices of this model are shown in Fig. 7; the models are shown superimposed on the imaging slices. In broad terms, the inversion imaging indicates the presence of the two anomalies and has a better resolution than Fig. 6.

4. Adding geological constraints to the inversion

A principal difficulty with the inversion of gravity data is the intrinsically non-unique results of any geophysical method based on a static potential field. Since the gravity field is known only at the surface of the earth, there are infinitely many equivalent density distributions beneath the surface that might produce the field. As a result, a meaningless inversion result is sometimes obtained. The addition of more known model parameters into the inversion process tends to produce a more

Fig. 7. Imaging slices at (a) y = 350 m; (b) y = 550 m; (c) z = 100 m; (d) z = 175 m. The color scales represent density.

Fig. 8. Constrained inversion imaging slices at (a) y = 350 m; (b) y = 550 m; (c) z = 100 m; (d) z = 175 m. The color scales represent density.
geologically meaningful inversion result and, at the same time, improve the image resolution.

From general knowledge of the geology of the region, and with well-log data, the rock density may be known at some given location. For each of the subsurface rectangular cells, it is then possible to define a density range \([\rho_{\text{min}}, \rho_{\text{max}}]\). After completing the iteration procedure, the result may be constrained as follows:

\[
\text{if}(\text{model} > \rho_{\text{max}})
\quad \text{model} = \rho_{\text{max}};
\]
\[
\text{else if}(\text{model} < \rho_{\text{min}})
\quad \text{model} = \rho_{\text{min}};
\]
\[
\text{else}
\quad \text{model} = \text{model};
\end{cases}
\]

In the present case, a density range restriction was added to the two-prism model following each iteration, limiting the rectangular cell density to fall within the range \([0,0.15]\). Fig. 8 shows the inversion slices, with the model superimposed as before, to illustrate the improved focusing effect and higher image resolution compared to that in Figs. 6 and 7.

5. Conclusion

This work presents an extension of the probability tomography imaging method for inversion of gravity anomaly data. The initial density model is provided by multiplying a small density value by the probability result, and then an iterative procedure is used to refresh the model until forward data and observed data fall within a given average error range. We have added inversion processing constraints for simple models to improve both the focusing effect and image resolution for cases in which the inversion result is not adequate. We have included two synthetic examples to demonstrate the effectiveness of the proposed method.

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