A new inversion method for \( (T_2, D) \) 2D NMR logging and fluid typing

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Abstract

One-dimensional nuclear magnetic resonance (1D NMR) logging technology has some significant limitations in fluid typing. However, not only can two-dimensional nuclear magnetic resonance (2D NMR) provide some accurate porosity parameters, but it can also identify fluids more accurately than 1D NMR. In this paper, based on the relaxation mechanism of \( (T_1, T_2) \) 2D NMR in a gradient magnetic field, a hybrid inversion method that combines least-squares-based QR decomposition (LSQR) and truncated singular value decomposition (TSVD) is examined in the 2D NMR inversion of various fluid models. The forward modeling and inversion tests are performed in detail with different acquisition parameters, such as magnetic field gradients \( G \) and echo spacing \( T_E \) groups. The simulated results are discussed and described in detail, the influence of the above-mentioned observation parameters on the inversion accuracy is investigated and analyzed, and the observation parameters in multi-\( T_2 \) activation are optimized. Furthermore, the hybrid inversion can be applied to quantitatively determine the fluid saturation. To study the effects of noise level on the hybrid method and inversion results, the numerical simulation experiments are performed using different signal-to-noise-ratios (SNRs), and the effect of different SNRs on fluid typing using three fluid models are discussed and analyzed in detail.

1. Introduction

Nuclear Magnetic Resonance (NMR) logging is a relatively new logging technology that has been used widely in the past decade due to its advantages for reservoir identification, component porosity calculation and fluid typing \((\text{George et al. (1998), Dunn et al. (2002), Chen et al. (2003)})\). NMR logging that only measures the transverse relaxation time \( (T_2) \) is known as one-dimensional NMR logging (1D NMR). When oil, gas and water coexist in a local porous medium, the transverse relaxation time \( (T_2) \) distributions of various fluids often overlap one another, so the usual differential spectrum method (DSM) and shift spectrum method (SSM) both have difficulty in distinguishing between oil, gas and water \((\text{Xiao (1998), George et al. (1998)})\). Addressing this difficulty led to the development of two-dimensional nuclear magnetic resonance (2D NMR) logging technology \((\text{Venkataramanan et al. (2002), Hurlimann et al. (2002), Dunn and Latorraca (1999)})\). However, 2D NMR logging tools and data acquisition methods require further development. The current 1D NMR logging cannot meet the two-dimensional observation requirements, so it is necessary to redesign its observation activations and the corresponding parameters. The new NMR tools, such as MReX of Baker Atlas and CMR-Scanner of Schlumberger, can both provide two-dimensional NMR logging capability. Dunn and Latorraca (1999), Sun et al. (2004), and Sun and Dunn (2004,2005a) of Chevron company and Hurlimann and Venkataramanan (2002) from Doll Research Center in Schlumberger have proposed a new method with an improved spin-echo pulse sequence (CPMG) using two windows, and they have achieved the transverse relaxation time \( (T_2) \) and diffusion coefficient \( (D) \) analysis of 2D NMR logging (known as \( (T_1, T_2) \) 2D NMR), which greatly improves the accuracy of fluid identification and saturation calculations. Sun and Dunn (2005b,c) and Sun et al. (2006) have studied a global inversion method for multi-dimensional NMR logging and have developed a method for 2D NMR inversion. Song et al. (2002) proposed a fast two-dimensional Laplace inversion for \( T_1-T_2 \) spectrum correction for \( (T_1, T_2) \) 2D NMR. Hursan et al. (2005) proposed a new NMR two-dimensional inversion for the \( T_1/T_2 \) app method for gas well petrophysical interpretation. In China, Xie et al. (2005,2007) have performed 2D NMR measurement experiments with man-made sandstone containing paramagnetic and natural shale sandstone...
saturated with water, and they have determined two-dimensional T2 distributions and internal magnetic field gradients (G), specifically (T2, G) 2D NMR. Their research yielded NMR measurements that are significant for the analysis of complex terrestrial sedimentary rock strata. Gu and Liu (2007) studied the use of traditional singular value decomposition (SVD) and improved the SVD algorithm for continuous (T2, D) 2D NMR inversion. Xie and Xiao (2009) studied the fluid typing of (T2, D) 2D NMR and (T1, T2) 2D NMR under different magnetic field gradients and signal-to-noise ratios (SNRs).

Because 2D NMR has not been used widely in oil field exploration, it is necessary to further study 2D NMR logging responses in different fluid models and to examine parameter designs for fluid typing. Based on the NMR relaxation mechanism under a magnetic field gradient, NMR logging mathematical theory with multiple spacing (T£) activation is studied in detail. The fluid models with different properties are designed and set up in advance. A hybrid inversion method is developed with the least-squares based QR decomposition (LSQR) and an improved truncated singular value decomposition (TSVD) algorithm. With this new inversion method, the (T2, D) 2D NMR spectrum maps of different fluid models are produced, and the influence on inversion results in different magnetic field gradients and different echo spacing groups that are compared and analyzed in detail. After the parameters are optimized, the saturation quantitative calculation tests are performed for the light oil, viscous oil and gas-water models, and the calculation accuracy is quite high, indicating that the hybrid inversion method is applicable for fluid typing and quantitative calculation of 2D NMR logging.

2. Theory and method

2.1. Two-dimensional NMR relaxation mechanism

With the terms of waiting time (T_W), echo spacing (T_e), and diffusion coefficient (D), NMR signals of CPMG can be written in Fredholm integral form:

\[ b(t, T_e, T_W) = \int f(D, D_e) k_1(t, T_e, D) dDdD_e + \varepsilon \]  \hspace{1cm} (1)

where, \( f(D, D_e) \) is the two-dimensional distribution of the proton number and \( k_1(t, T_e) \) and \( k_2(T_e, D) \) are the kernel functions under the conditions of the transverse relaxation time (T_e) and the diffusion coefficient (D), respectively, that is,

\[ k_1(t, T_e) = e^{-t/T_e} \]

\[ k_2(T_e, D) = e^{-1/12t^2G^2T_e^2D} \]

In the gradient field, the different pulse sequence responses of the CPMG with some echo spacings (T_e) are discretized to yield the following equation:

\[ b_{ik} = \sum_{j=1}^{m} \sum_{q=1}^{p} f_{ij} \exp \left( \frac{1 - 12t^2G^2T_e^2D_i}{T_e} \right) \exp \left( -t/T_e \right) + \varepsilon_{ik} \]  \hspace{1cm} (2)

where \( i = 1, \ldots, n_q; \quad k = 1, \ldots, q; \quad l = 1, \ldots, p; \quad j = 1, \ldots, m; \quad T_{ik} \) is the echo spacing of the kth echo-train; \( b_{ik} \) is the amplitude of the ith echo of the kth echo-train with the echo space of \( T_{ik} \); \( f_{ij} \) is the diffusion coefficient \((D_i)_{ij}\) is the gyromagnetic ratio; and \( G \) is the magnetic field gradient.

Eq. (2) can be expressed as the following equation:

\[ Ax = b \]  \hspace{1cm} (3)

where, \( A = \sum_{i=1}^{m} \sum_{j=1}^{p} f_{ij} \exp \left( -1/12t^2G^2T_e^2D_i \right) \exp \left( -t/T_e \right) \) and \( b \) is a vector composed of observation data \( b_{ik} \).

By solving the linear equations system (3) \( Ax = b \), the signal amplitude \((f_{ij})_0\) with transverse relaxation time \((T_{ik})_0\) and a diffusion coefficient \((D_i)_{0} \) can be calculated and the two-dimensional NMR spectrum can be determined.

2.2. Hybrid inversion method

Eq. (3), which is a least-squares problem, can be solved by double-diagonalization.

The LSQR algorithm was proposed to solve the least-squares problem by Paige and Saunders, 1982 and was based on QR decomposition and Lanczos iteration (Paige and Saunders (1982)). The algorithm can easily take advantage of the sparsity of the matrix to simplify the calculation, which is suited for solving large, sparse matrix problems (Moody and Xia (2004)).

The truncated singular value decomposition (TSVD) algorithm can determine the optimal answer through reducing the matrix A to some degree, but it loses the oscillatory component of the most powerful solution, so the algorithm reduces the resolution of the results and makes the spectrum discontinuous. To obtain a continuous spectrum, the optimal solution is determined by minimization of \( ||Ax - \Delta b|| \).

First, the LSQR algorithm is executed in the inversion problem and an initial solution is given as \( x_0 \) by the LSQR algorithm. Then, the TSVD algorithm is used for further inversion. During the TSVD inversion, the original equation is rewritten as \( A(x - x_0) = b - b_0 \), specifically, \( A(x - x_0) = \Delta b \). If the minimum value, \( \Delta x \), that provides the optimal solution to \( A\Delta x = \Delta b \) is obtained, the optimal solution, \( x = x_0 + \Delta x \), is the final result.

For the non-negative constraints, this method will not reduce the size of the matrix. If the solution components are less than zero, these components are directly changed to zero and then the changed solutions are reiterated until the components satisfy the non-negative conditions. Because the singular value decomposition process of the matrix \( A \) is executed only once, the amount of computation and the computing time are significantly reduced.

To verify the inversion precision, a parameter, \( R_e \), known as the relative error is constructed:

\[ R_e = \frac{\|x_{mod} - x_{inv}\|}{\|x_{mod}\|} \]

where, \( x_{mod} \) is the pre-designed model parameter and \( x_{inv} \) is the inverted model parameter.

3. Fluid properties and inversion tests

3.1. Fluid properties

The data kernel matrix for the inversion of the echo trains changes with changes in the echo spacing (T_e), the echo train number and the magnetic field gradient (G). The inversion results are studied under conditions of different echo spacing groups and different magnetic field gradients (G) in various fluid models. The simulation of oil, gas and water by the fluid models results in a change in the echo spacing (T_e) and diffusion coefficient (D) is divided into 30 bins, each of which is divided into 10 power exponents. The transverse relaxation time is between 1.0 ms and 10 s, and the diffusion coefficient ranges from 5.0 × 10^{-7} cm^2/s to 2.0 × 10^{-3} cm^2/s. The gyromagnetic ratio \((\gamma)\) is approximately 2π × 42.58 × 10^9 Hz/T.
3.2. Inversion tests

The waiting time \((T_W)\) is assumed to be long enough to ensure that the fluid in the formation is fully magnetized. The magnetic field gradient is assumed to be \(4 \times 10^{-3}\) T/cm. The \(T_E\) group is set to be \([7.2, 8.4, 9.6, 10.8, 12.0, 13.2]\) ms, and the echo number setting is similar to the dual-\(T_E\) activation of the MRIL-P logging (George et al. (1998)), that is, the acquisition time of each echo train is the same. The echo number of the short \(T_E\) echo train is large, and the echo number of the long \(T_E\) echo train is small. Therefore, the echo number of different echo trains is set to the integer of \(1440/T_E\). The number of iterations of the LSQR and the improved TSVD algorithms are both set to 800. The fluid model selected is the light oil model I, whose bound water and movable oil combined contents are 40%, and the movable water content is 60%, i.e., the oil saturation is 20% \((S_o=20\%)\).

The LSQR algorithm is first applied to the 2D NMR inversion test. Fig. 1 illustrates the inversion results using the LSQR algorithm. Fig. 1(a) shows the two-dimensional NMR spectrum in which the oil peak can be identified from the bound water and the movable water, but the water spectrum is not sufficiently distinguished. Fig. 1(b) shows the \(T_2\) distribution and \(D\) inverted comparisons with the model. The short \(T_2\) components \((1 < T_2 < 100\) ms) in the \(T_2\) distribution match well with those of the model, but the long \(T_2\) components \((100 < T_2 < 10,000\) ms) poorly match the data. By contrast, the fast diffusion components \((1.0 \times 10^{-3} < D < 0.5 \times 10^{-5}\) cm\(^2\)/s) inverted match well with those of the model, but the slow diffusion components \((0.5 \times 10^{-5} < T_2 < 1.0 \times 10^{-3}\) cm\(^2\)/s) match very poorly. Its running time is 16.67 s, and the relative error is 0.291.

Next, the TSVD algorithm is applied in 2D NMR inversion tests. Fig. 2 shows the inversion results using the TSVD algorithm. Fig. 2(a) shows that bound water, movable water and movable oil in the two-dimensional spectrum are more clearly distinguished than the same components as assessed by the LSQR algorithm in Fig. 1(a). In Fig. 2(b), the diffusion coefficient \((D)\) inverted matches well with the model, especially for the fast diffusion components, but the agreement is a slightly worse for the slow diffusion components. From the \(T_2\) distribution, the inversion result using TSVD is better than that using LSQR. The TSVD simulation running

<table>
<thead>
<tr>
<th>Fluids</th>
<th>Properties</th>
<th>Diffusion coefficient (cm(^2)/s)</th>
<th>(T_2) (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light oil</td>
<td></td>
<td>(4.0 \times 10^{-6})</td>
<td>600.0</td>
</tr>
<tr>
<td>Viscous oil</td>
<td></td>
<td>(8.0 \times 10^{-7})</td>
<td>100.0</td>
</tr>
<tr>
<td>Bound water</td>
<td></td>
<td>(4.0 \times 10^{-7})</td>
<td>20.0</td>
</tr>
<tr>
<td>Free water</td>
<td></td>
<td>(4.0 \times 10^{-8})</td>
<td>300.0</td>
</tr>
</tbody>
</table>

Fig. 1. Inversion results using the LSQR method. (a) 2D NMR spectrum; (b) Inversion results comparison.

Fig. 2. Inversion results using the TSVD method. (a) 2D NMR spectrum; (b) Inversion results comparison.
time is 10.79 s, and the relative error is 0.196. Therefore, the TSVD algorithm is more accurate and efficient than the LSQR algorithm. However, its accuracy requires further improvement.

Considering the comparison of the two inversion algorithms, can we take advantage of the two inversion algorithms? A hybrid approach utilizes both, where the LSQR algorithm is first used to invert, then the TSVD algorithm is executed with the LSQR inversion results input as the initial value of the TSVD inversion. A total of 400 iterations of both the TSVD and LSQR algorithms are used. Fig. 3 shows the inversion results by the LSQR–TSVD method. Fig. 3(a) shows that the two-dimensional spectrum matches well with the fluid model in three inversions, and Fig. 3(b) illustrates that the inverted $T_2$ distribution is in good agreement with that of the model for the short $T_2$ components as well as for the long $T_2$ components. The diffusion coefficient $D$ is in particularly good overall agreement with the model. The hybrid algorithm running time is 21.03 s, and the relative error is 0.0974. When the number of iterations for both the TSVD and LSQR sub-algorithms is set to 200, the hybrid algorithm operation time is 14.15 s and the relative error is 0.1058. Therefore, the LSQR–TSVD inversion method is more accurate than a single inversion algorithm. The inversion speed can be improved by reducing the number of iterations without significantly reducing the accuracy.

In the viscous oil–water model and the gas–water model, the 2D NMR inversion tests are performed using three inversion methods. The gas–water model I in Table 2 is chosen as an example with 20% gas saturation. When the iteration is set to 800 in the LSQR inversion, the relative error is 0.26596 and the running time is approximately 17.8438 s. When the iteration is set to 800 in the TSVD inversion, the relative error is 0.27960 and the running time is approximately 11.7813 s. For the hybrid inversion, when the iterations are both set to 100, the relative error is 0.10341 and the running time is 14.76 s. Fig. 4 illustrates the experiments in the gas–water model. Therefore, the hybrid inversion is also effective and fast for 2D NMR inversion and fluid typing in the gas–water models.

### Table 2

The properties of the fluid models with different saturation levels.

<table>
<thead>
<tr>
<th>Fluid models</th>
<th>Bound water (%)</th>
<th>Movable water (%)</th>
<th>Oil (%)</th>
<th>Gas (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light oil model I</td>
<td>20</td>
<td>60</td>
<td>20</td>
<td>–</td>
</tr>
<tr>
<td>Light oil model II</td>
<td>20</td>
<td>40</td>
<td>40</td>
<td>–</td>
</tr>
<tr>
<td>Light oil model III</td>
<td>20</td>
<td>20</td>
<td>60</td>
<td>–</td>
</tr>
<tr>
<td>Light oil model IV</td>
<td>20</td>
<td>0</td>
<td>80</td>
<td>–</td>
</tr>
<tr>
<td>Viscous oil model I</td>
<td>20</td>
<td>60</td>
<td>20</td>
<td>–</td>
</tr>
<tr>
<td>Viscous oil model II</td>
<td>20</td>
<td>40</td>
<td>40</td>
<td>–</td>
</tr>
<tr>
<td>Viscous oil model III</td>
<td>20</td>
<td>20</td>
<td>60</td>
<td>–</td>
</tr>
<tr>
<td>Viscous oil model IV</td>
<td>20</td>
<td>0</td>
<td>80</td>
<td>–</td>
</tr>
<tr>
<td>Gas model I</td>
<td>20</td>
<td>60</td>
<td>–</td>
<td>20</td>
</tr>
<tr>
<td>Gas model II</td>
<td>20</td>
<td>40</td>
<td>–</td>
<td>40</td>
</tr>
<tr>
<td>Gas model III</td>
<td>20</td>
<td>20</td>
<td>–</td>
<td>60</td>
</tr>
<tr>
<td>Gas model IV</td>
<td>20</td>
<td>0</td>
<td>–</td>
<td>80</td>
</tr>
</tbody>
</table>

### 4.2. Different magnetic field gradients

The waiting time ($T_{\text{cw}}$) is assumed to be long enough that the fluid in the formation is magnetized fully. The echo spacing group is set to [7.2, 8.4, 9.6, 10.8, 12.0, 13.2] ms, and the number of echoes is similar above. A series of echo trains with different echo spacing are simulated in the conditions of a hydrogen proton fully polarized in a given magnetic field gradient. Three different magnetic field gradients are used: $2.0 \times 10^{-3}$ T/cm, $3.0 \times 10^{-3}$ T/cm and $4.0 \times 10^{-3}$ T/cm.

First, a series of echo trains of the light oil model with different saturation are simulated and inversion experiments are executed. Fig. 5 shows the two-dimensional spectra and the comparison of the results with the one dimensional $T_2$ and $D$ in the light oil models. The two-dimensional spectra of bound water, movable water and movable oil are more readily distinguished with increasing magnetic field gradients. When the magnetic field gradient is $4.0 \times 10^{-3}$ T/cm, the two-dimensional spectrum of the three fluid components are most clearly distinguished (Fig. 4c) and the relative error ($R_e$) calculated is approximately 0.09733, which is the smallest relative error (Table 3). Similarly, some experiments for other fluid models with different oil saturation are also conducted, and the relative errors calculated are shown in Table 3. For each fluid model, the inversion accuracy is higher for larger magnetic field gradients.
Next, viscous oil models with different saturations are used to perform similar experiments. Fig. 6 shows the two-dimensional spectra and the corresponding one-dimensional $T_2$ and $D$ distributions. The bound water, movable water and oil components of the two-dimensional spectral map become sharper with increasing magnetic field gradient. However, the one-dimensional $T_2$ and diffusion coefficient ($D$) distributions are both in better agreement with the pre-designed model. Table 3 also shows the relative error calculated for the different light or viscous oil model, which indicates that the calculation error is reduced with increasing magnetic field gradient. For the viscous oil models with different saturations, the inversion results indicate a similar pattern. Therefore, the inversion precision is the highest for a magnetic field gradient of $4.0 \times 10^{-3} \text{T/cm}$.

Fig. 4. Comparison of the results of the three inversion methods for the gas–water model I ($S_o=40\%$). (a) LSQR algorithm; (b) TSVD algorithm; (c) LSQR–TSVD hybrid method.

Fig. 5. Two-dimensional spectra and comparison of the inversion results for different magnetic field gradients using light oil model II ($S_o=20\%$). (a) $G=2.0 \times 10^{-3} \text{T/cm}$; (b) $G=3.0 \times 10^{-3} \text{T/cm}$; (c) $G=4.0 \times 10^{-3} \text{T/cm}$.
In addition, Table 3 also shows that the inversion accuracy of different fluid models are different in a given magnetic field gradient and that the relative error is related to the oil saturation. For the fluid model containing light oil, bound water, and movable water, i.e., three-phase fluids existing in one reservoir, the inversion accuracy decreases with increasing oil saturation, which indicates a reducing ability to distinguish light oil from the other fluids. When there is no movable water in some reservoirs, there is only bound water and movable oil, thus the inversion accuracy is higher than that of the three-phase fluid models. For a fluid model containing viscous oil, the higher the oil saturation is, the lower the inversion accuracy is. In particular, when the oil saturation is 100%, the inversion accuracy decreases with increasing oil saturation, which indicates a reducing ability to distinguish light oil from the other fluids. When there is no movable water in some reservoirs, there is only bound water and movable oil, thus the inversion accuracy is higher than that of the three-phase fluid models. For a fluid model containing viscous oil, the higher the oil saturation is, the lower the inversion accuracy is. In particular, when the viscous oil and bound water coexist in the reservoir, the inversion accuracy is the lowest, which indicates the viscous oil and bound water cannot be distinguished from each other. In summary, movable water is more easily identified than movable oil by two-dimensional NMR inversion.

Similarly, some gas–water models are also used in the inversion tests at different magnetic field gradients $2.0 \times 10^{-3}$ T/cm, $3.0 \times 10^{-3}$ T/cm and $4.0 \times 10^{-3}$ T/cm. Fig. 7 shows the 2D NMR spectra and the comparison of $T_2$ and $D$. The relative errors are also listed in Table 3. From Fig. 7, the 2D NMR inversion results are identical and the relative errors in Table 3 exhibit no clear difference. Therefore, in 2D NMR logging for the gas–water models, the effect of the magnetic field gradient is weak.

4.3. Different echo spacing groups

When the inversion experiments with different echo spacing ($T_E$) groups are performed, with the fluids assumed to be fully polarized and the magnetic field gradient set to $4.0 \times 10^{-3}$ T/cm, the multiple echo spacing groups are set to [0.6,1.2,2.4,3.6,4.8,6.0] ms, [7.2,8.4,9.6,10.8,12.0,13.2] ms and [14.4,15.6,16.8,18.0,19.2,20.4] ms for the LSQR algorithm, the TSVD algorithm, and the LSQR–TSVD hybrid algorithm, respectively. The echo number of multi-$T_E$ echo trains is set up by the similar settings above, i.e., the entire acquisition time for each echo train is set to the same value.

First, a series of simulation experiments of ($T_E$, $D$) 2D NMR are performed for the light oil models with different saturations. Fig. 8 illustrates the inversion results with three echo spacing groups using the light oil model II ($S_o$=40%), in which the bound water, movable oil or water signal with the multi-$T_E$ group [7.2, 8.4, 9.6, 10.8, 12.0, 13.2] ms can be focused on more clearly. The one-dimensional $T_2$ distribution and the $D$ distribution are in good agreement with the pre-designed model (Fig. 8c), with the calculated relative error of approximately 0.08969, showing a higher precision than the components with the other multi-$T_E$ groups (see Table 4). Furthermore, the inversion results of the other fluid models with different oil saturations exhibit similar features, and their relative errors are listed in Table 4.

![Fig. 6. Two-dimensional spectra and comparison of the inversion results for different magnetic field gradients using viscous oil model II ($S_o$=60%).](image)

(a) $G=2.0 \times 10^{-3}$ T/cm; (b) $G=3.0 \times 10^{-3}$ T/cm; (c) $G=4.0 \times 10^{-3}$ T/cm.
To further examine the influence of the number of echo trains or multi-\(T_E\) groups on the inversion accuracy, the echo spacing ranging from 7.2 to 20.4 ms are divided into 8, 10, and 12 equal parts, i.e., the multi-\(T_E\) groups are \([7.2, 9.1, 11.0, 12.9, 14.7, 16.6, 18., 20.4]\) ms (8-\(T_E\)), \([7.2, 8.7, 10.1, 11.6, 13.1, 14.5, 16.0, 17.5, 18.9, 20.4]\) ms (10-\(T_E\)) and \([7.2, 8.4, 9.6, 10.8, 12.0, 13.2, 14.4, 15.6, 16.8, 18.0, 19.2, 20.4]\) ms (12-\(T_E\)), respectively. The numerical inversion results with 8-\(T_E\), 10-\(T_E\) and 12-\(T_E\) echo trains using light oil model III (\(S_w=60\%\)) are illustrated in Fig. 8(d)–(f), respectively, and the corresponding relative errors are listed in Table 4. The two-dimensional spectra are more readily distinguished with an increasing number of multi-\(T_E\) echo trains. The one-dimensional \(T_2\) distribution and the \(D\) distribution inverted with the 12-\(T_E\) group are in the best agreement with the model and have the highest the calculation accuracy. Thus, more accurate fluid typing is achieved when the number of multi-\(T_E\) echo trains increases gradually. When the bound water, free water and movable oil co-exist in the reservoirs, the inversion accuracy may decrease with increasing oil saturation. Only when bound water and movable oil are present is the accuracy for fluid typing higher in the two-phase fluid model than in the three-phase fluid model. This observation indicates that it is more difficult to distinguish oil from water when the fluid model is more complex. Therefore, in multi-\(T_E\) observation activation for the light oil model, the inversion results with hybrid echo spacing group cannot only identify bound water with fast \(T_2\) relaxation time and low diffusion coefficient but can also determine the components with slow \(T_2\) relaxation time and high diffusion coefficient in porous media. From the two-dimensional spectrum, light oil is easily distinguished from movable water and bound water.

Then, the viscous oil models with different saturations are also assessed using a similar series of tests with different echo spacing groups. Fig. 9(a)–(c) show the inversion results with three 6-\(T_E\) echo trains. From the two-dimensional spectrum inversion, the results with \([7.2,8.4,9.6,10.8,12.0,13.2]\) ms echo trains are also the best for the three 6-\(T_E\) echo trains inversion tests, and the \(T_2\) distribution and the \(D\) distribution of the inversion results also match well with the pre-designed model. The oil signal in Fig. 9(c) is more clearly distinguished from movable water and bound water than those in Fig. 9(a) and (b), and the relative error calculated is also the smallest, as shown in Table 4. Similarly, the echo trains are observed increase to 8-\(T_E\), 10-\(T_E\) and 12-\(T_E\) as shown in Fig. 9(d)–(f), respectively; the calculation errors are listed in Table 4. As shown in Table 4, for the viscous oil model, the inversion results are more accurate when the echo train observed increases, but when the oil saturation is extremely high or low, the calculation error approaches a minimum with an increase in the echo train. If the echo train continues to increase and exceeds a certain value, the accuracy will drop, which indicates that the water component is more sensitive than the oil component in the 2D NMR spectrum, so the water saturation should be selected first for the quantitative evaluation of the fluid in the reservoir.

Furthermore, the gas–water models with different gas saturations are also examined using a similar series of tests with different multi-\(T_E\) groups. Fig. 10 illustrates the 2D NMR inversion results with different \(T_E\) groups for gas–water model III, which has a gas saturation of 60%. From the inverted two-dimensional spectrum, the \((T_2, D)\) 2D NMR spectrum inverted with \([7.2, 8.4, 9.6, 10.8, 12.0, 13.2]\)ms echo trains is also the best of the three 6-\(T_E\) echo trains inversion tests, and the relative errors are also the smallest (see Table 5). In the multi-\(T_E\) inversion tests for the gas–water models, the echo spacing zone selected is from 7.2 to 13.2 ms, and the multi-\(T_E\) groups are \([7.2, 8.1, 8.9, 9.8, 10.6, 11.5, 12.3, 13.2]\) ms (8-\(T_E\)), \([7.2, 7.8, 8.4, 9.1, 9.7, 10.4, 11.0, 11.7, 12.5, 13.2]\) ms (10-\(T_E\)) and \([7.2, 7.7, 8.3, 8.8, 9.4, 9.9, 10.5, 11.0, 11.6, 12.1, 12.7, 13.2]\) ms (12-\(T_E\)). The experimental results for the gas–water models are shown in Fig. 10(d–f), and the calculation errors are listed in Table 5. From the relative errors in Table 5, the inversion results are more accurate when the observed echo train increases. The relative errors drop gradually as the gas saturation increases, which show that the gas component is more sensitive than the oil component in the 2D NMR spectrum, so the gas saturation is the first to be selected for the quantitative evaluation of the fluid in the reservoir.
Fig. 8. Two-dimensional spectra and comparison of the inversion results with different TE groups light oil model III \( (S_o = 60\%) \): (a) \( 6-TE \) from 0.6 to 6.0 ms; (b) \( 6-TE \) from 7.2 to 13.2 ms; (c) \( 6-TE \) from 14.4 to 20.4 ms; (d) \( 8-TE \) from 7.2 to 20.4 ms; (e) \( 10-TE \) from 7.2 to 20.4 ms; (f) \( 12-TE \) from 7.2 to 20.4 ms.

Table 4

Relative errors of the inversion results for the oil–water models with different \( T_E \) groups.

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<tbody>
<tr>
<td>Light oil model I ( (S_o = 20%) )</td>
<td>0.16145</td>
<td>0.09733</td>
<td>0.20924</td>
<td>0.06492</td>
<td>0.05058</td>
<td>0.03306</td>
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<tr>
<td>Light oil model II ( (S_o = 40%) )</td>
<td>0.24905</td>
<td>0.08969</td>
<td>0.16304</td>
<td>0.05329</td>
<td>0.03882</td>
<td>0.02693</td>
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<tr>
<td>Light oil model III ( (S_o = 60%) )</td>
<td>0.88541</td>
<td>0.07872</td>
<td>0.10472</td>
<td>0.03553</td>
<td>0.0268</td>
<td>0.01760</td>
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<tr>
<td>Light oil model IV ( (S_o = 80%) )</td>
<td>0.10897</td>
<td>0.099843</td>
<td>0.21116</td>
<td>0.06358</td>
<td>0.05164</td>
<td>0.03455</td>
<td></td>
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<tr>
<td>Viscous oil model I ( (S_o = 20%) )</td>
<td>0.27519</td>
<td>0.23321</td>
<td>0.24284</td>
<td>0.17865</td>
<td>0.15867</td>
<td>0.17677</td>
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<tr>
<td>Viscous oil model II ( (S_o = 40%) )</td>
<td>0.31652</td>
<td>0.25288</td>
<td>0.26211</td>
<td>0.29152</td>
<td>0.19321</td>
<td>0.10390</td>
<td></td>
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<tr>
<td>Viscous oil model III ( (S_o = 60%) )</td>
<td>0.36118</td>
<td>0.23717</td>
<td>0.33638</td>
<td>0.36621</td>
<td>0.22208</td>
<td>0.14086</td>
<td></td>
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<tr>
<td>Viscous oil model IV ( (S_o = 80%) )</td>
<td>0.71581</td>
<td>0.60035</td>
<td>0.67152</td>
<td>0.48135</td>
<td>0.54574</td>
<td>0.57450</td>
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The running time will be discussed for multi-TE echo train inversion. When the echo spacing is long, the total echo number will decrease because the entire acquisition time is stationary and the dimension number of the matrix $A$ will decrease, so the inversion speed will accelerate. Therefore, the inversion experiment with a long $T_E$ group has high speed.

5. Saturation quantitative calculation and analysis

The precision tests of the saturation quantitative calculation will be discussed for different fluid models. The multi-TE group is $\{7.2, 8.4, 9.6, 10.8, 12.0, 13.2\}$ ms, and the magnetic field gradient is $4.0 \times 10^{-3}$ T/cm. The light oil model II is chosen for an oil saturation of 40%, and the water content in the 2D NMR inversion initially calculated yields a water saturation of approximately 58.64%, so the oil saturation is approximately 41.36%. The relative error is approximately 3.4%. Similarly, for viscosity model II ($S_o = 40\%$), the water content is also initially calculated and the oil saturation is subsequently calculated to have a value of approximately 41.69%. The relative error is approximately 4.23%.

In the gas–water models, the gas saturation is also calculated through 2D NMR inversion by the hybrid method. For gas–water model II, the gas saturation is assumed to be 40% and is also initially calculated to have a value of approximately 40.15%. The relative error is approximately 3.9%.
Fig. 10. Two-dimensional spectra and comparison of the inversion results with different TE groups using gas–water model III (S_g = 60%). (a) 6-TE from 0.6 to 6.0 ms; (b) 6-TE from 7.2 to 13.2 ms; (c) 6-TE from 14.4 to 20.4 ms; (d) 8-TE from 7.2 to 13.2 ms; (e) 10-TE from 7.2 to 13.2 ms; (f) 12-TE from 7.2 to 13.2 ms.

Table 5
Relative errors of the inversion results for gas–water models with different TE groups.

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<tbody>
<tr>
<td>Gas model I (S_g = 20%)</td>
<td>0.27157</td>
<td>0.10341</td>
<td>0.19430</td>
<td>0.11746</td>
<td>0.11656</td>
<td>0.11960</td>
<td></td>
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<tr>
<td>Gas model II (S_g = 40%)</td>
<td>0.29099</td>
<td>0.18924</td>
<td>0.36624</td>
<td>0.12333</td>
<td>0.12659</td>
<td>0.11730</td>
<td></td>
</tr>
<tr>
<td>Gas model III (S_g = 60%)</td>
<td>0.22285</td>
<td>0.12388</td>
<td>0.49026</td>
<td>0.11262</td>
<td>0.10700</td>
<td>0.10211</td>
<td></td>
</tr>
<tr>
<td>Gas model IV (S_g = 80%)</td>
<td>0.17070</td>
<td>0.10703</td>
<td>0.51460</td>
<td>0.10503</td>
<td>0.08964</td>
<td>0.08628</td>
<td></td>
</tr>
</tbody>
</table>
The quantitative calculation tests above indicate that the hybrid inversion method is suited for the quantitative evaluation of fluids and for calculation accuracy. The calculation accuracy in the light oil and gas–water models is better than that in the viscous oil models.

6. Analysis of noise effects

First, the light oil model II is chosen to study the effects on the inversion results of \((T_2, D)\) 2D NMR, which has bound water saturation, movable water saturation, and movable oil values of approximately 20%, 40% and 40%, respectively. The magnetic field gradient is assumed to be \(3.0 \times 10^{-2} \text{T/cm}\), and the echo spacing group is set to \([7.2, 8.7, 10.1, 11.6, 13.1, 14.5, 16.0, 17.5, 19.0, 20.4]\) ms. In the experiment, the different SNRs are added into the synthesized echo strains and the inversion is performed by the hybrid inversion method.

When the SNR is 150, the inversion results are illustrated in Fig. 11(a), and the light oil can be clearly distinguished from

![Fig. 11. Inversion results for light oil–water models with different SNRs. (a) SNR=150; (b) SNR=100, \(z=5.0\times10^{-7}\); (c) SNR=50, \(z=0.003\); (d) SNR=25, \(z=0.6\).](image-url)
the bound and movable water in the two-dimensional spectrum and the one-dimensional $T_2$ and $D$ distributions. Therefore, the inversion process may be directly executed by the hybrid inversion method when the SNR exceeds 150. Furthermore, the relative error ($R_e$) is approximately 0.0995, and the oil saturation, bound water saturation, movable saturation, and water saturation are approximately 40.21%, 19.99%, 39.80%, and 59.79%, respectively.

Fig. 12. Inversion results for viscous oil–water models with different SNRs. (a) SNR = 150; (b) SNR = 100, $\alpha = 5.0e-7$; (c) SNR = 50, $\alpha = 0.004$; (d) SNR = 25, $\alpha = 0.4$. 

When the SNR is relatively low (for example, less than 150), the damping factor \( a \) should be added into the hybrid inversion, i.e., the inversion problem \( Ax = b \) is first changed as the solution of \( (A^TA + aI)x = A^Tb \) from Eq. (3), then the hybrid inversion method is used to solve the rewritten equation. Different SNRs, such as 100, 50 and 25, are chosen for similar simulation experiments, and the inversion results are illustrated in Fig. 11(b–d). In the inversion, the corresponding damping factors are approximately 5.0e–7, 0.003, and 0.6, i.e., the damping factors should increase as the SNRs decrease, and the relative errors are approximately 0.3563,

![Fig. 13. Inversion results for gas–water models with different SNRs. (a) SNR = 150; (b) SNR = 100, \( a = 1.0e-6 \); (c) SNR = 50, \( a = 0.003 \); (d) SNR = 25, \( a = 0.175 \).](image-url)

0.5703 and 0.7634, respectively. From Fig. 11(b and c), when the SNR is 50 or 100, two-dimensional spectrum, as well as one-dimensional $T_2$ distribution and diffusion coefficients, can directly be used to identify the oil and water. When the SNR is lower than 50, the oil and water are mixed, as shown in Fig. 8(d), and it is difficult to accurately distinguish them. The water saturation is approximately 0.5979, 0.5607, 0.5435 and 0.4635 in four SNRs. It is noted that the fluid typing is not more accurate in the light oil–water model with a decreasing SNR.

Next, similar experiments are also performed for the viscous oil–water model. Based on the inversion results in Fig. 12, when the SNR is 150, bound water, heavy oil, and movable water can be clearly distinguished from one another. When the SNR is 100 or 50, the three fluid components are also still clearly identified. When the SNR is lower than 50, the viscous oil and movable water peaks begin to overlap, and the bound water peak is also not tightly focused. The relative errors are approximately 0.2469, 0.4182, 0.5921, 0.8153 for the four SNRs, i.e., the quantitative results are not more accurate with decreasing SNR. In addition, the inversion results are not more accurate for the viscous oil–water model compared to the light oil–water model.

In the gas–water model, similar experiments are also performed to study the effects of SNRs on the inversion results. As in the other simulations, when the SNR exceeds 150, the damping factor is not required in the inversion; whereas the damping factor is required when the SNR is lower than 150. When the SNR exceeds 100, gas, movable water and bound water can be distinguished from one another, but when the SNRs are decreasing, the gas and movable water peaks gradually overlap. When the SNR is approximately 25, the gas and movable water peaks are fully mixed together and cannot be accurately identified (see Fig. 13).

7. Conclusions and discussion

The following conclusions resulted from the numerical simulations and the analysis of a series of experiments:

1. In 2D NMR logging inversion, a number of experiments indicate that the LSQR inversion algorithm can distinguish short $T_2$ high diffusion components, whereas the TSVD inversion algorithm can identify long $T_2$ and slow diffusion components. Therefore, the hybrid inversion method is produced by combining the LSQR and TSVD algorithms. Furthermore, the accuracy of the hybrid inversion is improved even for a small number of iterations, and the calculation speed is faster than that of the single TSVD or LSQR algorithms.

2. In the fluid models with oil and water, for multi-$T_2$ observation with the same waiting time, the fluid typing is more accurate when the magnetic field gradient is slightly larger.

3. From a number of 2D NMR inversion tests, the multi-echo trains can be optimized to clearly distinguish oil or gas from bound water and free water. With an increase in echo spacing, the inversion accuracy increases gradually in the light oil or gas–water models. However, for the viscous oil fluid models, the inversion accuracy reaches a maximum, and the inversion accuracy decreases for higher echo spacing numbers past that for the peak accuracy.

4. For complex fluid models with bound water, movable water and movable oil, the water component of the 2D NMR spectrum is more sensitive and more clearly distinguished. If the oil saturation is the same, when the reservoirs only contain bound water and movable oil without free water, the light oil can be identified more accurately than viscous oil. For the gas–water models, the gas component of the 2D NMR spectrum is more sensitive and can be more clearly identified.

5. The numerical simulation experiments of the $(T_2, D)$ 2D NMR using different fluid models indicate that the reservoir fluids can be qualitatively determined according to the location and amplitude of the focused signals of the two-dimensional spectrum. In three fluid models, the quantitative calculation results using the light oil and gas–water models are more accurate than those using the viscous oil models.

6. The effect of SNR on the $(T_2, D)$ 2D NMR inversion results using three fluid models is different. When the SNR is higher than 150, the hybrid inversion method may directly be used for the inversion of $(T_2, D)$ 2D NMR, but when the SNR is relatively lower (SNR < 150), the damping factor should be added into the hybrid inversion. The accuracy of the inversion results can also be improved by adjusting the damping factor. The inversion results are not more accurate with decreasing SNRs. In the light oil–water and the viscous oil–water models, oil, movable water, and bound water can be distinguished from one another when the SNR is over 50, whereas these three fluid components cannot be accurately identified with an SNR of 25. Furthermore, the inversion results are more accurate using the light oil–water model than those using the viscous oil–water model. In the gas model, the three fluid components can be distinguished from one another when the SNR is over 100, whereas they cannot be accurately identified when the SNR is lower than 100. Therefore, the effect of SNR in the oil–water model is lower than that in the gas–water model.

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Appendix A. Supporting information

Supplementary data associated with this article can be found in the online version at http://dx.doi.org/10.1016/j.cageo.2012.07.030.

References


