A Semianalytical Model for Horizontal-Well Productivity With Pressure Drop Along the Wellbore

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Summary
This study aims to develop a semianalytical model to calculate the productivity index (PI) of a horizontal well with pressure drop along the wellbore. It has been indicated that by introducing novel definitions of horizontal-well permeability and conductivity, the equation of fluid flow along a horizontal well with pressure drop has the same form as the one for fluid flow in a varying-conductivity fracture. Thus, the varying-conductivity-fracture model and PI model can be used to obtain the PI of a horizontal well. Results indicate that the PI of a horizontal well depends on the interaction between horizontal-well conductivity, penetration ratio, and Reynolds number. New type curves of the penetration ratios with various combinations of parameters have been presented. A complete-penetration zone and a partial-penetration zone can be identified on the type curves. Based on the type curves, two examples have also been presented to illustrate the advantages of this work in optimizing parameters of horizontal wells.

Introduction
PI is an important parameter for potential estimation of production. The PI for a horizontal well is well-documented in literature. On the basis of the assumption of an infinite-conductivity wellbore, several methods have been presented to predict the steady-state PI of a horizontal well in an infinite reservoir, such as the Borisov (1964) method, the Giger-Reiss-Jourdan (Giger 1987) method, the Joshi (1988) method, and the Renard and Dupuy (1991) method. In addition, a few models focused on the pseudosteady-state PI for a closed boundary (Mutalik et al. 1988; Babu and Odeh 1989; Goode and Kuchuk 1991; Chen et al. 2005; Hagoort 2009). However, these models ignored the effect of pressure drop along the wellbore caused by friction loss when calculating PI. As a result, the PI from those studies increases monotonically with increasing the length of a horizontal wellbore. Many studies have revealed that the pressure loss along the horizontal wellbore cannot be neglected, especially for long horizontal wells with high production rates (Dikken 1990; Novy 1995; Ozkan et al. 1995, 1999; Cho and Shah 2001, 2002).

Three methods have been reported to incorporate the pressure drop into the mathematical model of PI. The first method is to directly add a wellbore-pressure drop to the hypothetical infinite-conductivity well-pressure drop (Joshi 1991; Cho 2003; Adesina et al. 2011; Owusu et al. 2015). This is a simple and approximate method without knowing flux distribution along the wellbore. The second method is to implement the distribution of infinite-conductivity specific productivity (influx per pressure drop) to compute the wellbore-pressure drop caused by friction (Dikken 1990; Novy 1995). Dependent on the assumptions of steady-state flow in the reservoir and constant PI per unit of wellbore length, the Dikken (1990) correlation overestimated the effect of the friction with rough walls (Ozkan et al. 1995; Cho and Shah 2001, 2002; Cho 2003). These approximate models did not rigorously couple the wellbore and reservoir, which may lead to unrealistic conclusions on the effect of wellbore hydraulics. Ozkan et al. (1995, 1999) presented a comprehensive model that couples wellbore and reservoir hydraulics for the single-phase flow of a slightly compressible liquid. The Ozkan method (Ozkan et al. 1995, 1999) is more mathematically rigorous, and can account for different flow regimes (laminar and turbulent) and friction loss.

Using the PI, the length of the horizontal well can be optimized. Novy (1995) provided criteria to select a reasonable horizontal length. Penmatcha et al. (1997) developed a semianalytical model for homogeneous reservoirs that can quantify the effects of both single-phase oil and two-phase oil/gas-flow pressure loss in the wellbore on the overall well performance. Cho (2003) stated that the production is no longer proportional to the horizontal-well length after an inflection point. Adesina et al. (2011) further extended the Cho (2003) work and proposed an improved predictive model taking into account the effects of the friction loss, kinetic change, and fluid accumulation.

Although many models have been proposed to obtain the PI of a horizontal well, there is a lack of a rigorous model to calculate the PI and reveal the mechanisms of fluid flow along a horizontal well with pressure drop under both laminar and turbulent flows. In the presented study, a semianalytical solution for the horizontal-well PI with pressure drop along the wellbore has been developed. By spatially varying horizontal-well conductivity with respect to the wellbore hydraulic effect, a general horizontal-well-flow model is developed, which has the same form as the fracture-flow model (Cinco-Ley et al. 1978; Luo and Tang 2015). Thus, the method developed for varying-conductivity fractures (Luo and Tang 2015) can be used to calculate the horizontal-well PI directly. With the new semianalytical method, the effects of interaction between horizontal-well conductivity and Reynolds number on the dimensionless PI are discussed, and new type curves are also presented for optimizing the horizontal-well parameters.

Mathematical Models
Basic Assumptions. Fig. 1 presents the schematic of a horizontal well in a closed rectangular reservoir. The details of the physical model are elaborated as follows:
- No-flow boundary condition is applied in the lateral directions (x- and y-direction) and impermeable boundaries at the top (z = 0) and bottom (z = h). It is homogeneous with constant porosity ($\phi$) and permeability ($k$).
- The flow in the reservoir is assumed to be single phase, isothermal, and slightly compressible, with constant compressibility ($c$), viscosity ($\mu$), and density ($\rho$).
• The horizontal wellbore extends, horizontally, in the x-direction and is located at elevation (z_w) from the bottom boundary of the reservoir. The wellbore is of length (L_h), radius (r_w), and roughness (ε) on its surface.

• The well is produced at the heel under constant rate (q), whereas no flow is assumed across the toe of the well. The fluid that converges into the wellbore from the reservoir is assumed to be continuous along the wellbore.

Mathematical models are presented in terms of dimensionless variables for fluid flow in the reservoir and horizontal wellbore (Appendix A).

Derivation of Flow Model Along the Horizontal Well. Under the assumption of 1D steady-state flow in the horizontal wellbore (Ozkan et al. 1995), we have

$$\frac{dp_h}{dx} = C_E \frac{\rho}{\pi^2 r_w^4} f \cdot q_h^2,$$  

(1)

with $C_E = 9.117 \times 10^{-13}$, and where $f$ denotes the Fanning friction factor. The Reynolds number is defined as

$$N_{Re} = \frac{C_E \cdot q_h \cdot \rho}{\mu_w}, N_{Re} = 6.157 \times 10^{-2},$$  

(2)

Substituting Eq. 2 into Eq. 1 and implementing the dimensionless transform, we have

$$\frac{C_E \times 2 \times \pi^2 \times 141.2 \cdot \frac{r_w^2}{C_E} \cdot \frac{A_w}{kh_{tot}} \cdot \frac{dph}{dx}}{f N_{Re}} = -q_h \cdot 2\pi,$$  

(3)

where $A_w$ denotes the cross-sectional area of the wellbore.

We define the horizontal-well permeability, $k_h(x)$, as

$$k_h(x) = \frac{C_E \times 2 \times \pi^2 \times 141.2 \cdot \frac{r_w^2}{C_E} \cdot \frac{A_w}{kh_{tot}}}{f N_{Re}} = 1.88227 \times 10^{14} \times \left( \frac{r_w^2}{f N_{Re}} \right).$$  

(4)

In this study, the flow regime in the wellbore may be laminar, transitional, or turbulent, depending on the Reynolds number. For the sake of simplicity, we assume two flow regimes that are laminar ($N_{Re} < 2,300$) and turbulent ($N_{Re} > 2,300$). Furthermore, the Fanning friction factor can be calculated as

$$f = \frac{1}{1.14 - 2 \log \left( \frac{d}{d} + \frac{21.28}{N_{Re}^{0.9}} \right)^{0.5}},$$  

(5)

for turbulent flow, and

$$f = 16/N_{Re},$$  

(6)

for laminar flow.

Eq. 4 can be further written as

$$k_h(x) = \frac{1.88227 \times 10^{14} r_w^2}{16} \times \left( \frac{16}{f N_{Re}} \right) \left[ \frac{(f N_{Re})_0}{(f N_{Re})_0} \right] = k_w \times \frac{16}{f N_{Re}} \left( \frac{f N_{Re}}{(f N_{Re})_0} \right)_0 = k_w \times \left( \frac{f N_{Re}}{(f N_{Re})_0} \right)_0 \times \left( \frac{(f N_{Re})_0}{(f N_{Re})_0} \right) \times \left( \frac{(f N_{Re})_0}{(f N_{Re})_0} \right),$$  

(7)

where

$$k_w = 1.17642 \times 10^{13} \cdot r_w^2.$$  

(8)

Note that Fanning friction factor $f$ is the function of Reynolds number ($N_{Re}$), which is proportional to the cross-sectional flux ($q_h$).

Thus, the horizontal-well permeability $k_h(x)$ is a function of $q_h$, and changes spatially along the horizontal wellbore.

For the laminar flow in the wellbore, substituting Eq. 6 into Eq. 7 yields

$$k_h(x) = \frac{1.88227 \times 10^{14} r_w^2}{16} = 1.17642 \times 10^{13} \cdot r_w^2.$$  

(9)
Eq. 9 shows that the horizontal-well permeability is a constant for laminar flow along the wellbore. The horizontal-well conductivity can also be defined as

$$C_{HD}(x) = \frac{k_w(x)A_c}{kh \ell_{ext}} = C_{HD} \times \left[ \frac{16}{(f N_{Re})_1 \cdot (f N_{Re})_0} \right]$$

where $C_{HD}$ is the dimensionless horizontal-well conductivity defined by Ozkan et al. (1995).

$$C_{HD} = \frac{k_w A_c}{kh \ell_{ext}} \quad \text{Note that the horizontal-well conductivity, } C_{HD}(x), \text{ is not constant but a function of position. Substituting Eqs. 7 and 10 into Eq. 3, the following equation for a horizontal well in dimensionless form is obtained:}$$

$$C_{HD}(x_D) \frac{dp_{HD}}{dx_D} = -2\pi \cdot q_{wD}, \quad \text{with boundary conditions}$$

$$\left( \frac{dp_{HD}}{dx_D} \right)_{x_D=0} = -\left( \frac{2\pi}{C_{HD}} \right), \quad 0 \leq x_D \leq 2, \quad \left( \frac{dp_{HD}}{dx_D} \right)_{x_D=2} = 0.$$ 

Note that Eq. 12 has the same form as that for a varying-conductivity fracture (Luo and Tang 2015). We define a dimension transformation (Luo and Tang 2015) as follows:

$$\tilde{\xi}_D = \tilde{\xi}_D(x_D) = C_{HD}^{-1} \int_0^{x_D} \frac{dx_D}{C_{HD}(x_D)}$$

where

$$\hat{C}_{HD} = 1 \int_0^2 \frac{dx_D}{C_{HD}(x_D)} \quad x_D \in [0, 2].$$

Note that $\hat{C}_{HD}$ is constant.

Substituting Eq. 15 into Eq. 12 and integrating Eq. 12 with the boundary conditions, we have (Luo and Tang 2015)

$$p_{wD} - p_{HD}(\tilde{\xi}_D) = \left( \frac{2\pi}{\hat{C}_{HD}} \right) \left[ \tilde{\xi}_D - \sum_{i=1}^{N} \tilde{\xi}_{Di} \Delta q_{Di} \right].$$

**Calculation of PI.** We discretize the horizontal well into $N$ segments for calculating the PI of the horizontal well. According to the definition of the dimensionless PI (Luo et al. 2016), we have

$$J_D = \left( 141.2BRL \frac{q}{kh} \right) \cdot \left( \frac{q_{Di}}{p_{avg} - p_w} \right) = \sum_{i=1}^{N} q_{Di} \cdot \frac{\Delta q_{Di}}{p_{avg} \cdot \ell_{avg} \cdot \ell_{ext}} = \sum_{i=1}^{N} J_{Di}.$$ 

For the flow from the reservoir into the horizontal wellbore, each segment can be considered as a uniform-flux horizontal well. According to the superposition principle, the dimensionless pressure at the $i$th segment in the drainage area can be written as (Ozkan 1988)

$$p_{wD} = p_{avg D} + \sum_{j=1}^{N} q_{Di} \cdot F_j(x_{Di}, y_{Di}, x_{wDj}, y_{wDj}, \ell_{Di}, \ell_{avg D}), \quad i = 1, 2, ..., N.$$ 

For the flow in the horizontal wellbore, Eq. 17 can be written into the discretization form (Luo and Tang 2015):

$$p_{HD} - p_{wD} = \left( \frac{2\pi}{\hat{C}_{HD}} \right) \left[ \tilde{\xi}_{Di} \sum_{j=1}^{N} q_{Di} - \frac{(\tilde{\xi}_{Di} - \tilde{\xi}_{Di-1}/2)^2}{2 \cdot \Delta q_{Di}} \cdot q_{Di} \right]$$

In addition, the pressure and the flux must be continuous along the horizontal well:

$$\tilde{p}_{HD} = \tilde{p}_D; \quad \tilde{q}_{wD} = \tilde{q}_D.$$
Eqs. 19, 20, and 21 can be merged together to obtain equations of PI, which can be further solved with the Gaussian elimination method. Note that an iterative procedure will be used to achieve the stable distribution of conductivity, $C_{ad}(x_d)$, in Eq. 15 (Luo and Tang 2015).

**Results and Discussions**

**Model Validation.** To the best of the authors’ knowledge, there are no examples of horizontal-well PI in a closed rectangular reservoir that take into account the effect of wellbore-pressure drop. Hagoort (2009) presented an approximate solution for an infinite-conductivity horizontal well. In addition, Ozkan et al. (1999) claimed that the differences between horizontal-well and vertical-fracture responses become negligible for a large value of penetration ratio, and the performance of an infinite-conductivity horizontal well approaches that of an infinite-conductivity vertical fracture for large penetration ratios. Therefore, to verify our method, here we calculate the PI for an infinite-conductivity horizontal well (black solid line) and compare it with the results for an infinite-conductivity horizontal well from Hagoort (2009) (blue circle with cross) and for an infinite-conductivity fracture from Ozkan et al. (1999) (red dots) (Fig. 2).

![Comparison of dimensionless PI for an infinite-conductivity horizontal well.](image)

It is shown that our results agree well with the Hagoort (2009) approximate solutions when $I_x$ is less than 0.5 (Fig. 2). For large penetration ratios ($I_x > 0.5$), our results deviate from the Hagoort (2009) approximate solution, but are consistent with the Ozkan et al. (1999) infinite-conductivity-fracture solution.

**Effect of Reynolds Number on the Dimensionless PI.** In this subsection, the results calculated by the new semianalytical method are presented. Taking into account the relatively weak effect of well eccentricity on the pressure responses (Ozkan et al. 1999), we assume that the well is halfway between the top and bottom boundaries ($z_{wp} = 0.5$). According to the dimensionless definitions, it is found that variables of horizontal-well conductivity ($C_{ad}$), dimensionless length ($L_D$), penetration ratio ($I_x$), and boundary length ($x_d, y_d, z_d$) are not independent. In the following discussions, we will illustrate the influence of combined variable $C_{ad0} \times L_D, I_x, N_{Ref}$, and $\varepsilon_D$ on the dimensionless PI in a rectangular reservoir.

The effect of the Reynolds number on the dimensionless PI has been presented in Figs. 3 through 6. For a relatively small value of $C_{ad0} \times L_D$, 10^5, when the $L_D$ varies from 1 to 150, the corresponding penetration ratio ($I_x$) is between 6.67 x 10^{-1} and unity (Table 1). A small value of $I_x$ means a short horizontal wellbore with high horizontal-well conductivity, whereas a large value of $I_x$ denotes low horizontal-well conductivity. Thus, the maximum horizontal-well conductivity is 10^5 corresponding to the $I_x$ of 6.67 x 10^{-1} and the minimum horizontal-well conductivity is 66.7 at $I_x = 1$ (Table 1).

It is shown that the Reynolds number exerts a strong effect on the dimensionless PI ($J_D$) (Fig. 3). For an extremely small value of dimensionless penetration ratio ($I_x < 0.07$) [i.e., short horizontal well with high conductivity ($C_{ad}$)], the effect of $N_{Ref}$ on $J_D$ can be ignored. As stated previously, larger $C_{ad}$ indicates a higher contrast in the conductivities of wellbore and reservoir. The pressure drop along the wellbore is negligible compared with that in the reservoir, and the response of the system is governed by the conductivity of the reservoir (Ozkan et al. 1999).

For a longer horizontal well, its contact area with the reservoir increases with respect to a larger penetration ratio. Meanwhile, flow resistance in the wellbore also increases, which yields a negative effect on the PI. As a result, the PI of a horizontal well depends on the balance of these two opposing factors: the length and the resistance (Adesina et al. 2011). With the increase of penetration ratio ($I_x$), some different behaviors for the laminar and turbulent flows can be noticed. For laminar flow, such that $N_{Ref} = 1,000$, $J_D$ increases with the increase of the penetration ratio ($I_x$), meaning that the positive factor (increasing length) prevails over the PI.

For turbulent flow ($N_{Ref} > 2,300$), an inflection point of $J_D$ can be found. Beyond the inflection point, $J_D$ declines with the increase of the penetration ratio ($I_x$), which indicates that the negative factor (flow resistance) becomes pronounced for a longer horizontal wellbore. Note that the inflection point of $J_D$ moves toward the shorter wellbore (smaller $I_x$) and becomes less obvious as $N_{Ref}$ increases.
Fig. 4 displays the distribution of dimensionless PI along the horizontal well at $I_x = 0.5$ and $C_{ord} L_D = 10^4$ for different $N_{Re}$ values. It is shown that $J_{Di}$ is distributed as an asymmetric U-shaped curve with respect to the midpoint of the wellbore. For each segment, $J_{Di}$ decreases with the increase of the Reynolds number. If the Reynolds number ($N_{Re}$) is very large, most of the fluid entering the wellbore is from the segments close to the heel of the well ($N_{Re} = 10^6$, pink squares).

In Figs. 5 and 6, we examine the total $J_D$ in the heel of wellbore and distributions of $J_{Di}$ along the wellbore at a relatively high value of $C_{ord} L_D = 10^6$. The dimensionless conductivity of the horizontal well magnifies 100 times, from 6,667 to $10^6$. Comparing Fig. 5 with Fig. 3, no significant difference is noticed for the correlation between $I_x$ and $J_D$. However, the effect of the Reynolds number on $J_D$ is weakened for a higher horizontal-well conductivity. First, the maximum difference of $J_D$ is reduced from 2.76 ($3.48 - 0.72 = 2.76$) (Fig. 3) to 1.52 ($3.95 - 2.43 = 1.52$) (Fig. 5). Second, for $I_x$ less than 0.38, the curves overlap, indicating a negligible influence of the Reynolds number on the $J_D$. Third, the inflection point is not present on curves with a Reynolds number less than $10^5$, which means that the positive effect of longer horizontal wells on $J_D$ prevails. Last, comparing Fig. 6 with Fig. 4, profiles of $J_{Di}$ get close to each other for different Reynolds numbers, and approximately symmetric U-shaped curves with respect to the midpoint of the wellbore can be obtained.
Effect of the Surface Roughness on the Dimensionless PI. The effects of surface roughness on the dimensionless PI at $N_{Re}t = 10^3$ (laminar flow) and $N_{Re}t = 10^6$ (turbulent flow) are presented in Figs. 7 and 8, respectively. To make a comparison, we set three values of dimensionless surface roughness ($0, 10^{-4}/C_0$, and $10^{-3}/C_0$) to illustrate their effects on $JD$. As seen from Figs. 7 and 8, the dimensionless surface roughness exerts no effect on $JD$ under the condition of laminar flow (the curves overlap) for $N_{Re}t = 10^3$. For turbulent flows, the effect of dimensionless surface roughness on $JD$ will be more significant for larger values of $ChD_0/L_D$ (Figs. 7 and 8). Moreover, with the decrease of surface roughness, $JD$ increases more dramatically for a larger $ChD_0/L_D$ ($10^6$) (Fig. 8) than for a smaller value ($10^4$) (Fig. 7).

Applications. Type Curves of Penetration Ratio. Using the new model, we calculated 2,500 cases to obtain the PI of parameter combinations with different penetration ratios, Reynolds numbers, and conductivities. We take the penetration ratio ($I_x$) corresponding to the maximum PI as the objective optimization parameter for each case. We define two variables, $X$ and $Y$, as

$$X = f(\nu, \mu, q, r_w) = \log(N_{Re}) = \log(\nu/\mu) + \log(q/r_w) - 1.21, \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad 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Type curves with \( X \), \( Y \), and optimal \( I_x \) are drawn in Fig. 9. For given values of \( X \) and \( Y \), an optimal \( I_x \) can be obtained from Fig. 9.

The optimal length of horizontal well (OLHW) can be calculated by multiplying reservoir length \( x_e \) with the optimal \( I_x \). It is found that two zones (complete-penetration zone and partial-penetration zone) can be identified.

- At the complete-penetration zone (yellow zone) corresponding to small \( X \) (low production rate, Eq. 22) and large \( Y \) (low permeability or thin pay, Eq. 23), a long horizontal well can be used to achieve the maximum PI.
- For a given value of \( Y \), OLHW decreases with increase of value \( X \) for a high production rate.
- For a fixed \( X \), OLHW increases with the increase of value \( Y \) through low-permeability or thin reservoir, which indicates that long horizontal wells are suitable for the development of thin and low-permeability reservoirs.
- The wellbore radius exerts strong effect on OLHW. The increase of wellbore radius will result in a small value of \( X \) and a large value of \( Y \), which both lead to a large penetration ratio (Fig. 9).

**Case Study.** Two examples with different parameters have been presented to illustrate the application of type curves to optimize the horizontal-well parameters.

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<td>125.00</td>
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<tr>
<td>90</td>
<td>3.33</td>
<td>1.67</td>
<td>6.00 \times 10^{-1}</td>
<td>111.11</td>
</tr>
<tr>
<td>100</td>
<td>3</td>
<td>1.5</td>
<td>6.67 \times 10^{-1}</td>
<td>100</td>
</tr>
<tr>
<td>110</td>
<td>2.73</td>
<td>1.36</td>
<td>7.33 \times 10^{-1}</td>
<td>90.91</td>
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<tr>
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<td>1.25</td>
<td>8.00 \times 10^{-1}</td>
<td>83.33</td>
</tr>
<tr>
<td>130</td>
<td>2.31</td>
<td>1.15</td>
<td>8.67 \times 10^{-1}</td>
<td>76.92</td>
</tr>
<tr>
<td>140</td>
<td>2.14</td>
<td>1.07</td>
<td>9.33 \times 10^{-1}</td>
<td>71.43</td>
</tr>
<tr>
<td>150</td>
<td>2</td>
<td>1</td>
<td>10,000</td>
<td>66.67</td>
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</tbody>
</table>

Table 1—The relationship of horizontal-well parameters. \( x_{ID} \times L_D = 300 \), \( y_{ID} \times L_D = 150 \), \( z_{ID} = 0.5 \), \( f_{ID} = 6.64 \times 10^{-3} \), \( C_{ID} \times L_D = 10^6 \).

**Fig. 7—**Effect of surface roughness on the dimensionless PL for \( N_{ref} = 10^3 \) (laminar flow) and \( N_{ref} = 10^6 \) (turbulent flow) at \( C_{ID} \times L_D = 10^6 \).
Case 1: Optimization of Horizontal-Well Parameters for a Reservoir with High Permeability.

The basic parameters for Case 1 are listed in Table 2. We set the values of production rate between 100 and 20,000 STB/D. Using the parameters, we calculate the value of $X$, $Y$, $N_{Re}$, $I_x$, and OLHW, as listed in Table 3. The values of optimal penetration ratios are marked with a green square in Fig. 10. As seen from Table 3, OLHW depends on the production rate of the horizontal well. A shorter horizontal well with high production rate can be used for this case.

![Diagram](image-url)

**Fig. 8**—Effect of surface roughness on the dimensionless PI for $N_{Re} = 10^3$ (laminar flow) and $N_{Re} = 10^6$ (turbulent flow) at $C_{fD} = 10^6$.

![Diagram](image-url)

**Fig. 9**—Type curves with $X$, $Y$, and optimal penetration ratio ($I_x$).

**Table 2**—Basic parameters for a high-permeability reservoir.

<table>
<thead>
<tr>
<th>Basic Model Parameters</th>
<th>Value</th>
<th>Basic Model Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir pressure (psi)</td>
<td>2,100</td>
<td>Oil density (lbm/ft$^3$)</td>
<td>55</td>
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<tr>
<td>Porosity (%)</td>
<td>25</td>
<td>Wellbore radius (ft)</td>
<td>0.25</td>
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<td>Permeability (md)</td>
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<td>Reservoir length (ft)</td>
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<td>Net pay (ft)</td>
<td>37.65</td>
<td>Reservoir width (ft)</td>
<td>5,647</td>
</tr>
<tr>
<td>Volume factor (RB/STB)</td>
<td>1.1</td>
<td>Total compressibility (1/psi)</td>
<td>6x10$^{-6}$</td>
</tr>
<tr>
<td>Well location ($z$) (ft)</td>
<td>18.83</td>
<td>Fluid viscosity (cp)</td>
<td>1.15</td>
</tr>
</tbody>
</table>

Table 2—Basic parameters for a high-permeability reservoir.
Case 2: Optimization of Horizontal-Well Parameters for a Reservoir With Low Permeability. The basic parameters for Case 2 are listed in Table 4.

Using the method presented previously, the OLHW can be calculated as listed in Table 5. The values of optimal penetration ratios are marked with a blue solid circle in Fig. 10. Compared with the case of the high-permeability reservoir (Tables 2 and 3, Fig. 10), it is found that all the calculated values of $X$ and $Y$ fall into the complete-penetration zone (Fig. 10, blue solid circle). It also shows that the effects of pressure drop along the horizontal well can be neglected for low-permeability reservoirs. A complete penetration is recommended for this case.
Conclusions
Several conclusions can be drawn from this work, as follows:
1. Using a new definition of horizontal-well conductivity, the fluid flow in the horizontal wellbore can be considered as a varying-conductivity flow. The method formulated for the flow in a varying-conductivity fracture (Luo and Tang 2015) can be used here to deal with the flow inside the horizontal wellbore with pressure drop.
2. The PI ($J_D$) depends on the interaction of horizontal-well conductivity ($C_{hd}$), penetration ratio ($I_x$), and Reynolds number ($N_{Re}$). Results show that a large $J_D$ can be achieved with a long horizontal well (large $I_x$) under laminar flow (low $N_{Re}$). With the increase of the Reynolds number, the flow resistance inside the wellbore becomes pronounced for long horizontal wells, which results in an inflection point on the $J_D$ curve.
3. The type curves of optimal penetration ratios ($I_x$) with different parameter combinations have been constructed. A complete-penetration zone and a partial-penetration zone can be found on the type curves. Two examples have been used to illustrate the application of the presented model for horizontal-well parameter optimization. It is shown that the value of the optimal penetration ratio ($I_x$) will entirely fall into the complete-penetration zone for a low-permeability reservoir with a low production rate. Thus, a long horizontal well can be drilled to enhance the PI. In contrast, for a high-permeability reservoir, an optimal penetration ratio might be found in the partial-penetration zone. The optimal length of a horizontal well depends on the production rate. Smaller optimal horizontal-well lengths will be achieved with larger production rates.

Nomenclature

- $A_c =$ cross-sectional area, ft$^2$
- $B =$ volume factor, RB/STB
- $c_t =$ total compressibility, psi$^{-1}$
- $C_{hd} =$ dimensionless horizontal-well conductivity
- $f =$ Fanning friction factor
- $h =$ net-pay, ft
- $J =$ PI, B/D/psi
- $J_D =$ dimensionless PI
- $k =$ permeability of drainage area, md
- $k_h =$ horizontal-well permeability, md
- $L_h =$ length of horizontal well, ft
- $L_{ref} =$ reference length, ft
- $N =$ discretized segments of horizontal well
- $N_{Re} =$ Reynolds number at any position along the horizontal wellbore
- $N_{Re,t} =$ Reynolds number at the heel
- $p =$ pressure, psi
- $p_{avg} =$ average pressure, psi
- $p_h =$ pressure in the horizontal well, psi
- $p_i =$ initial formation pressure, psi
- $p_w =$ pressure at the heel, psi
- $q =$ production rate, STB/D
- $q_{hc} =$ flow rate in cross-sectional area, STB/D
- $q_{MD} =$ dimensionless flow rate
- $r_w =$ wellbore radius, ft
- $x =$ coordinate in the $x$-direction, ft
- $x_e =$ length of drainage area, ft
- $x_f =$ fracture half-length, ft
- $x_w =$ dimensionless wellbore coordinate in the $x$-direction
- $y =$ coordinate in the $y$-direction, ft
- $y_e =$ width of drainage area, ft
- $y_w =$ dimensionless wellbore coordinate in the $y$-direction
- $z =$ coordinate in the $z$-direction, ft
- $z_w =$ well location in the vertical interval, ft
- $\varepsilon =$ surface roughness, ft
- $\mu =$ fluid viscosity, cp
- $\phi =$ porosity, fraction

Subscripts

- $D =$ dimensionless
- $h =$ horizontal-well property
- $i =$ initial or segment $i$
- $t =$ total
- $w =$ wellbore property

Acknowledgments
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References


### Appendix A—Dimensionless Definitions of Variables

For the sake of simplicity, we will present our solution in terms of the following dimensionless variables. The dimensionless reservoir pressure $p_\text{D}$, dimensionless average reservoir pressure $p_\text{avgD}$, and dimensionless wellbore pressure $p_\text{wD}$ are given, respectively, by

$$p_\text{D} = \frac{k h (p_i - p_w)}{141.2 \mu B q}, \quad p_\text{avgD} = \frac{k h (p_i - p_\text{avg})}{141.2 \mu B q}, \quad p_\text{wD} = \frac{k h (p_i - p_w)}{141.2 \mu B q}.$$

The dimensionless PI is

$$J_\text{D} = \left(\frac{141.2 B \mu}{kh}\right) \cdot \left(\frac{q}{p_\text{avg} - p_w}\right) = \left(\frac{141.2 B \mu}{kh}\right) \cdot J.$$

The dimensionless distances $x_\text{D}$, $y_\text{D}$, $r_\text{D}$, $r_{wD}$, $z_\text{D}$, and $z_{wD}$ are defined, respectively, by

$$x_\text{D} = \frac{x}{L_{\text{ref}}}, \quad y_\text{D} = \frac{y}{L_{\text{ref}}}, \quad r_\text{D} = \frac{r}{L_{\text{ref}}}, \quad r_{wD} = \sqrt{(x_\text{D} - 1)^2 + y_\text{D}^2}, \quad r_{wD} = \frac{r_w}{h}.$$

and

$$z_\text{D} = \frac{z}{h}, \quad z_{wD} = \frac{z_w}{h}.$$

The dimensionless length is

$$L_\text{D} = \frac{L_h}{2h}.$$

The dimensionless formation thickness and horizontal-well length are

$$h_{\text{D}} = \frac{h}{L_{\text{ref}}}, \quad L_{\text{D}} = \frac{L_h}{L_{\text{ref}}}.$$
The dimensionless horizontal-well conductivity is

\[
C_{D}(x) = \frac{k_h(x)A_c}{kh_{\text{ref}}}.
\]  
(A-7)

Note that the definition of dimensionless horizontal-well conductivity is different from that in the Ozkan et al. (1995, 1999) model. According to our definition, the dimensionless horizontal-well conductivity is not a constant but changes along the wellbore.

The relative well surface roughness is

\[
e_{D} = \frac{e}{2r_{w}}.
\]  
(A-8)

The dimensionless rate \(q_{hD}\) and dimensionless cross-sectional rate \(q_{hcD}\) are defined as

\[
q_{hD} = \frac{q_h}{q}, \quad q_{hcD} = \frac{q_{hc}}{q} = \int_{0}^{L_h} q_h(x', t)dx'/q.
\]  
(A-9)

In this paper, we define the reference length \(L_{\text{ref}}\) as

\[
L_{\text{ref}} = \frac{L_h}{2}.
\]  
(A-10)

Thus

\[
h_{D}L_{D} = 1.
\]  
(A-11)

The penetration ratio of the horizontal well is defined as

\[
I_\epsilon = L_h/x_c.
\]  
(A-12)

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**SI Metric Conversion Factors**

- \( \text{bbl} \times 0.1589874 \) \( \text{m}^3 \)
- \( \text{cp} \times 0.001 \) \( \text{Pa} \cdot \text{s} \)
- \( \text{ft} \times 0.3048^* \) \( \text{m} \)
- \( \text{ft}^2 \times 0.0929 \) \( \text{m}^2 \)
- \( \text{psi} \times 6.894757 \) \( \text{kPa} \)

*Conversion factor is exact.

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