Bidimensional empirical mode decomposition (BEMD) for extraction of gravity anomalies associated with gold mineralization in the Tongshi gold field, Western Shandong Uplifted Block, Eastern China

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A B S T R A C T

A bidimensional empirical mode decomposition (BEMD) program on a MATLAB platform was effectively used to handle gravity signals for the Tongshi gold field. This yielded a two-dimensional intrinsic mode function (IMF3) image that meticulously depicts the spatial distribution relationship between various gold deposits and the different geological units of the gold field. By combining the IMF3 image with geological features, the Tongshi gold field can be divided into three geological units. Unit I is made up of both Archean greenstone and Cambrian–Ordovician carbonate rocks and is defined as a concealed basement that exhibits weak negative and positive gravity anomalies. Unit II is defined as a Mesozoic volcanic sediment basin consisting of Jurassic and Cretaceous volcanic-sedimentary rocks, which developed on the concealed basement and exhibits an obvious negative gravity anomaly. Unit III, located on the concealed basement on the SW side of the Mesozoic volcanic-sedimentary basin, can be subdivided into subunits IIIa and IIIb. The former, with a negative circular gravity anomaly, indicates that the Tongshi complex consists of both Yanshan diorite-porphyrite and syenite-porphyry. Unit IIIb, with a positive ring gravity anomaly distributed around a negative circular gravity anomaly, indicates a possible contact metasomatic mineralization zone around the Tongshi complex. Almost all types of gold deposits are located within this contact metasomatic mineralization zone and thus the zone is a prospective area for gold deposits.

1. Introduction

Geoscience data usually exhibit nonlinearity and nonstationarity because of the complexities of geological processes and the heterogeneous composition and texture of geological bodies (Cheng, 1995, 1999, 2003, 2004, 2006, 2008; Li and Cheng, 2004). Traditional methods, such as Fourier transforms and geostatistics, are not suitable for nonlinear and nonstationary processes (Lovejoy et al., 2005; Chen et al., 2006; Huang et al., 1998; Huang, 2006). To handle this problem, Huang et al. (1998) developed an adaptive method for analysis of nonlinear and nonstationary data, named the Hilbert–Huang transform (HHT) that comprises empirical mode decomposition (EMD) and Hilbert spectral analysis.

Our study focused on bidimensional empirical mode decomposition (BEMD). For one-dimensional data, a complicated data set can be decomposed into a finite number of intrinsic mode functions (IMFs). These 1D IMFs can be obtained from a series of sifting processes. Since Huang established the 1D EMD, it has been used in many fields, such as seismic signal analysis, noise filtering, airborne gravity data and hydrological and environmental time series analysis (Huang et al., 1998, 2001; Flandrin et al., 2004; Hassan and Peirce, 2008; Rao and Hsu, 2008). Later, Nunes et al. (2003, 2005) developed EMD for 2D image texture analysis, called BEMD. BEMD can be used for texture analysis and estimation of the instantaneous frequencies of an image (Shen et al., 2005).

The BEMDs for image texture analysis is only suitable for dealing with continuous raster data in 2D but is not well suitable for dealing with scattered data in 2D such as exploration geophysical data (e.g. gravitational data, magnetic data) and exploration geochemical data.

In the present study, we developed a BEMD method for analysis of scattered data in 2D through dealing with the gravitational data surveyed at a scale of 1:50,000 for the Tongshi gold field, Western Shandong Uplift Terrain, Eastern China; and investigated the spatial relationship between deep geological structures and gold mineralization in the Tongshi gold field by extracting gravitational information at a certain frequency to provide valuable evidence for predicting the location of deep gold deposits.
2. BEMD principle

A complicated data set can be adaptively decomposed into a finite number of components of different frequencies called IMFs using an iterative sifting process that continues until the number of extrema is ≤ 2 (one maximum and one minimum). The method can be used to extract local high-frequency oscillations from the original data set. IMFs have the following features: (a) each IMF has the same number of zero crossings and extrema; (b) at any point, the mean value of the upper envelope defined by the local maxima and the lower envelope defined by the local minima tends to zero (Huang et al., 1998).

BEMD is similar to one-dimensional EMD but extremum detection and surface interpolation of envelopes are more complicated.

Assume that Ori(m, n) is an original data set that can be decomposed into a finite number of bidimensional intrinsic mode functions (BIMFs) showing the texture of different frequencies (scales) using a 2D sifting process. From high to low frequencies, Ori(m, n) can be decomposed into a series of BIMFs and a residual

\[
\text{Ori}(m, n) = \sum_{i=1}^{t} B_i(m, n) + \text{Res}(m, n).
\]

where \( B_i(m, n) \) is the ith BIMF component and \( \text{Res}(m, n) \) is the residual. In general the BIMF\(_i\) frequency is higher than that of any other BIMF within the same segment.

Various types of filters can be designed, such as high-pass \( S_{hp} \), band-pass \( S_{bp} \) and low-pass \( S_{lp} \) filters. Specific BIMFs can also be chosen as the filters according to requirements

\[
S_{hp}(m, n) = \sum_{i=1}^{k} B_i(m, n),
\]

\[
S_{bp}(m, n) = \sum_{i=k}^{p} B_i(m, n),
\]

\[
S_{lp}(m, n) = \sum_{i=p}^{t} B_i(m, n) + \text{Res}(m, n).
\]

Specific BIMF components that contain useful geological information can be chosen as the filtering result for original geoscience data. Thus, a BEMD scheme can be used for decomposition of a gravity data set and extraction of gravitational anomalies associated with gold mineralization.

3. BEMD approach

The BEMD procedure is similar to one-dimensional EMD. A BIMF component is obtained using a bidimensional sifting process in which neighboring windows are used to detect the extrema. A multiquadric (MQ) method can be used to perform the surface interpolation of envelopes.

3.1. Extremum detection

The extrema (maxima and minima points) can be established by comparing neighboring window values. The value of each extremum is strictly higher or lower than the values of its neighbors \( (M_1 \sim M_8) \).

3.2. Bidimensional envelope and interpolation

The powerful MQ scheme was used for bidimensional enveloped interpolation. Since MQ was first introduced by Hardy (1971, 1990) for interpolation of geographical surfaces and gravitational and magnetic anomalies, it has been widely applied in geosciences and other fields (Franke, 1982; Carr and Fright, 1997; Carr et al., 2001; Yao et al., 2002; Zhang and Zhou, 2007).

The MQ method is a global interpolation scheme for scatter data using a radial basis function (RBF) (Buhmann, 2004). For the scatter data \( (x_i, y_j) \) (i.e. the extrema), the MQ method uses equations of the following form:

\[
H(X) = p(X) + \sum_{j=1}^{n} \lambda_j \Phi(X_j), \quad j = 1, 2, \ldots, n,
\]

where \( H(X) \) is the value of the extremum \( (x_i, y_j) \), \( p(X) \) is a low-order polynomial \( (p(X) = c_0 + c_1 x + c_2 y, \text{or zero here}) \), \( \lambda_j \) is the parameters to be determined, \( \Phi = [(x_i - x_j)^2 + R^2]^{1/2} \) and \( R^2 \) is an interpolation constant that is discussed later.

Thus, Eq. (5) in matrix notation is

\[
\begin{bmatrix}
A & P \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\lambda \\
C
\end{bmatrix} =
\begin{bmatrix}
H \\
0
\end{bmatrix}.
\]

where

\[
A = \Phi(|X_i - X_j|) = \begin{bmatrix}
d_{11} & d_{12} & \cdots & d_{1j} & \cdots & d_{1n} \\
ed_{21} & d_{22} & \cdots & d_{2j} & \cdots & d_{2n} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
d_{n1} & d_{n2} & \cdots & d_{nj} & \cdots & d_{nn}
\end{bmatrix},
\]

\[
P = \begin{bmatrix}
1 & x_1 & y_1 \\
1 & x_2 & y_2 \\
\vdots & \vdots & \vdots \\
1 & x_n & y_n
\end{bmatrix},
\]

\[
\lambda = [\lambda_1, \lambda_2, \ldots, \lambda_n]^T, \quad C = [c_0, c_1, c_2]^T \quad \text{and} \quad H = [H_1, H_2, \ldots, H_n]^T.
\]

In Eq. (6), only the matrices \( \lambda \) and \( C \) are unknown and

\[
\begin{bmatrix}
A & P \\
0 & 0
\end{bmatrix}^{-1}
\begin{bmatrix}
H \\
0
\end{bmatrix}.
\]

is a real symmetric matrix. Thus, the solution for the parameters is

\[
\begin{bmatrix}
\lambda \\
C
\end{bmatrix} =
\begin{bmatrix}
A & P \\
0 & 0
\end{bmatrix}^{-1}
\begin{bmatrix}
H \\
0
\end{bmatrix}.
\]
Thus, the formula for calculating an interpolated value at point \( I \) is:

\[
\begin{bmatrix}
H_I \\
0
\end{bmatrix} = \begin{bmatrix}
A_I & P_I \\
P^T_I & 0
\end{bmatrix}^{-1} \begin{bmatrix}
H \\
0
\end{bmatrix}.
\] (8)

Hence, upper or a lower envelope interpolation can be accomplished.

The next step is to discuss the influence of the constant \( R^2 \). The accuracy of the MQ scheme depends on the selection of \( R^2 \), so an optimal \( R^2 \) is very important. Hardy (1971, 1990), Foley (1987), Carlson and Foley (1991) discussed how to choose an appropriate \( R^2 \) value for calculation of \( d_{ij} \).

There are many methods for calculating \( R^2 \) (Hardy, 1971, 1990; Franke, 1979). In the present study the following formula was used (Franke, 1979)

\[
R^2 = \frac{1.25D^2}{\sqrt{n}},
\] (9)

where \( D \) is the diameter of the circle bounding the data and \( n \) is the number of data points. The value of \( R^2 \) is thus essentially related to the number of data points and to their distribution in the \( x-y \) plane. However, Carlson and Foley (1991) found that the optimal \( R^2 \) value is strongly influenced by the data point values. Kansa (1990a, 1990b) found that the value of \( R^2 \) can control the shape of the basis function. Flat sheet-like basis functions are usually associated with large \( R^2 \), bowl-like functions with intermediate \( R^2 \), and narrow cone-like functions with small \( R^2 \). For rapidly varying or highly oscillatory data values, \( R^2 \) is usually very small.

### 3.3. Stopping criterion

For each BIMF, a stopping criterion should be determined during the sifting process. According to Huang (2006), this can be accomplished by limiting the size of the standard deviation (SD), which can be computed from the two consecutive sifting results as follows (Huang, 2006; Nunes et al., 2003, 2005):

\[
SD_d = \sum_{m=1}^{m=n} \left( \frac{|h_{m-1}(m,n)-h_{m}(m,n)|}{h_{m-1}^2(m,n)} \right)^2.
\] (10)

The 2D sifting process can be influenced by the SD. The number of BIMFs increases as SD decreases and the computing time increases significantly. The mean and SD of the mean matrix mean\((m,n)\) for upper and lower envelopes should also be considered when determining the stopping criterion.

### 3.4. BEMD algorithm and procedure

Similar to the 1D EMD of Huang (2006), the BEMD algorithm was first developed by Nunes et al. (2003, 2005). In the present study, the envelope interpolation for this algorithm has been improved. The 2D sifting process algorithm is defined as follows (Fig. 2).

If Ori\((m,n)\) is the 2D data set to be decomposed:

1. Initialization: \( r_0(m,n) = \text{Ori}(m,n) \) and \( j = 1 \) is the BIMF index.
2. Extraction of the \( j \)-th BIMF component
   (i) \( h_0(m,n) = r_{j-1}(m,n), \ i = 1 \).
   (ii) Detect all the extrema (maximum or minimum) of \( h_{i-1}(m,n) \).
   (iii) Compute the upper (lower) envelope: upper\(_{i-1}(m,n)\) and lower\(_{i-1}(m,n)\).
   (iv) Calculate the envelope mean: mean\(_{i-1}(m,n)\) = \( [\text{upper}_{i-1}(m,n) + \text{lower}_{i-1}(m,n)]/2 \).
   (v) \( h_i(m,n) = h_{i-1}(m,n) - \text{mean}_{i-1}(m,n) \).
3. Residual \( r_r(m,n) = r_{j-1}(m,n) - s_i(m,n) \).
4. For \( j = j + 1 \) repeat steps (ii) and (iii) until the number of extrema for \( r_{j}(m,n) \) is less than 2.

### 3.5. BIMF orthogonality

Huang et al. (1998) investigated the orthogonality of 1D IMFs. Orthogonality can be defined for any two IMFs, \( C_j \) and \( C_k \). The formula for measuring orthogonality is

\[
\text{IO}_{jk} = \text{abs} \sum_{t} \frac{C_j(t)C_k(t)}{C_j^2(t) + C_k^2(t)},
\] (11)

where \( C_j \) and \( C_k \) are 1D IMFs. The IO value usually depends on the signal length, as well as the decomposed results. For a 1D EMD method, the IO value should typically be < 1%. For extremely
short data, the IO value could be as high as 5%. Orthogonality is satisfied in a practical sense, but it is not yet guaranteed theoretically. Although all IMFs decomposed using EMD are not orthogonal, the IMFs still have a physical meaning for some specific signals. In addition, orthogonality is a requirement only for linear decomposition systems (Huang et al., 1998).

4. Application

Geophysical fields are very useful in inferring deep-seated geological structures and delineating concealed geological objects such as buried intrusive bodies and ore bodies. Effective use of gravity fields, like other geophysical fields, depends on establishment of a set of signatures that characterize forms, sizes and depths, as well as masses of various geological objects and their relationship to mineralization (Pan and Harris, 2000). The most direct information acquired from gravity fields is the density of geological bodies. A high gravity value indicates the presence of geological objects with higher average density than the materials surrounding them. Conversely, a low gravity value indicates the presence of geological bodies with relatively low average density.

Because of heterogeneity in the density of geological bodies created during complicated geological processes, even the same lithological unit in different spatial locations can cause different gravity anomalies, whereas different lithological units can produce similar gravity fields. This non-unique correspondence can cause some difficulties in inferring deep-seated geological structures and in delineating concealed geological objects. Thus, intrinsic geological and geochemical information is required.

The scale of gravity anomalies is related not only to the size, but also to the depth of geological bodies. The same scale and type of anomaly might be produced by different lithological units located at different depths. Different scales and types of anomalies are possibly associated with differences in both the lithology and buried depth of geological bodies. These complexities and difficulties mean that new information decomposition techniques are required to identify possible ore-bearing locations from huge amounts of geoscience data.

4.1. Geology and mineralization of the Tongshi gold field

The Tongshi gold field is located in the concealed basement area on the southwestern margin of the Mesozoic Pingyi volcanic-sedimentary basin in the Western Shandong Uplifted Block, Eastern China. The concealed basement area where Cambrian–Ordovician carbonates lie on top of Archean greenstone was intruded by the Tongshi subvolcanic alkaline intrusive complex consisting of syenite porphyries and diorite porphyrites and constituting the primary ore-controlling factor in the gold field. Numerous gold deposits (occurrences) scattered around the Tongshi complex can be classified as: (a) porphyry gold located within the Tongshi intrusive complex, (b) Skarn iron–copper–gold located in the inner contact metasomatic zone between the intrusive complex and its host Cambrian–Ordovician carbonate rocks, (c) Crypto-breccia gold deposits and (d) Carlin-type gold deposits located in the outer contact metasomatic zone between the intrusive complex and its host rocks. Thus, the various types of gold deposits constitute a gold mineral resource series (Fig. 3) (Chen et al., 2000, 2001).

Two $^{40}\text{Ar}/^{39}\text{Ar}$ analyses of amphiboles from the diorite porphyrite and the syenite porphyrite suites revealed ages of 189 and 188 Ma, respectively (Lin and Tan, 1996). Zircon SHRIMP U–Pb ages for the intrusive complex vary from 167.9 to 183 Ma (Hu et al., 2005). The intrusive complex exhibits obvious negative gravity fields (Fig. 4).

4.2. Gravity anomaly associated with gold mineralization in the Tongshi gold field

Gravity data surveyed for a grid of 500 m × 250 m were provided by the Yanzhou Geology & Mineral Exploration Institute of Shandong province. The resolution of the survey data is
The density of exposed geological bodies was 2.73–2.90 g/cm³ for the Archean greenstone belt, 2.64–2.76 g/cm³ for Cambrian–Ordovician carbonates, 2.61–2.71 g/cm³ for the Tongshi intrusive complex and 2.46–2.53 g/cm³ for Jurassic–Cretaceous volcanic-sedimentary rocks (Wang et al., 2003).

The original gravity image (Fig. 4) was decomposed into five 2D IMFs (BIMF₁, BIMF₂, BIMF₃, BIMF₄ and BIMF₅) and residuals Res(m,n) by BEMD to determine the relationship between the geological structure at depth and the spatial distribution of gold deposits

\[
\text{Ori}(m,n) = \sum_{i=1}^{5} \text{BIMF}_i(m,n) + \text{Res}(m,n),
\]

where Ori(m,n) are the original 2D gravity data; BIMF(m,n) are the 2D IMFs and Res(m,n) are the 2D residual for zero extrema. The 2D IMFs represent 2D gravity data of different frequency
Although \( \text{BIMF}_1, \text{BIMF}_2, \text{BIMF}_3, \text{BIMF}_4 \) and \( \text{BIMF}_5 \) decrease in frequency, the frequency of \( \text{BIMF}_i \) is higher than that of \( \text{BIMF}_{i+1} \) only within the same range, and not in any range.

It is assumed that \( SD = 2 \) for decomposition of the gravity data surveyed for a grid of 500 × 250 m. The 1D decomposition results for the northeastern (line AB) and northwestern directions (line CD) of the Tongshi gold field at a mutually perpendicular angle (Fig. 4) are shown in Figs. 5 and 6, respectively. The smoothness of the curves increases as the frequency decreases from \( \text{IMF}_1 \) to \( \text{IMF}_5 \).

The orthogonality of the BIMFs was checked in both directions (Table 1). The results show that almost all the IO values are close to zero and thus orthogonality is approximately satisfied.

**5. Discussion and conclusions**

Both \( \text{IMF}_3 \) and \( \text{IMF}_5 \) components among the five IMFs mentioned above have geological significance (Figs. 7 and 8). The \( \text{IMF}_5 \) image in Fig. 7 is a low-pass-filtered component of the original gravity images. Two basic geological units at depth within the study area are evident. One is a concealed basement with a double layer comprising Cambrian–Ordovician carbonate rocks laying on Archean greenstone, characterized mainly by a positive gravity anomaly (I). The other is a Mesozoic volcanic-sedimentary basin that consists of Jurassic and the Cretaceous volcanic-sedimentary rocks, with an obvious negative gravity anomaly (II).

The \( \text{IMF}_3 \) image in Fig. 8 is a band-pass-filtered component of the original gravity data, which clearly reveals the geological structure of the Tongshi gold field. The Tongshi gold field includes three basic geological units according to Fig. 8. (a) A concealed basement (I), with medium-intensity gravity anomalies is present, for which high-intensity positive gravity anomalies along the NW trend located between units II and III may be Archean metamorphic basement swells. (b) The Mesozoic volcanic-sedimentary basin (II) that was developed on the concealed basement exhibits an obvious negative gravity anomaly extending in the NW direction. (c) The Tongshi intrusive complex (III) intruded into the concealed basement and developed at the southwestern margin of the Mesozoic volcanic-sedimentary basin. According to its gravity anomaly features, unit III can be subdivided into subunits IIIa and IIIb. The circular negative gravity anomaly with an obvious negative anomalous center (Illia) might reflect the spatial distribution of the Tongshi intrusive complex. The positive ring gravity anomaly around IIIa is Illb, which might represent the contact metasomatic zone that developed between the Tongshi intrusive complex and its host rocks. The spatial distributions of various types of gold deposits are controlled by unit III (Fig. 8).

Taking a geological section along line EF in Fig. 8 yields a geological–geophysical pattern for the Tongshi gold field showing the formation and distribution of gold deposits (Fig. 9). During 168–189 Ma the Tongshi intrusive complex (Illia), with a negative gravity anomaly varying from –2 to –265 \( \mu \text{m/s}^2 \), intruded into the concealed basement, where Archean greenstone was covered by Cambrian–Ordovician carbonate rocks. The gravity anomaly of the concealed basement (I) generally varies from –100 to –2 \( \mu \text{m/s}^2 \), but in some segments this rapidly increases from –2 to 246 \( \mu \text{m/s}^2 \), possibly reflecting upwelling zones of the Archean basement. The contact metasomatic zone usually displays positive gravity anomalies, which vary from 0 to 160 \( \mu \text{m/s}^2 \) (Illb). This can be

<table>
<thead>
<tr>
<th>Section AB</th>
<th>Section CD</th>
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<tbody>
<tr>
<td>IO(_{12})</td>
<td>0.1008</td>
</tr>
<tr>
<td>IO(_{13})</td>
<td>0.1065</td>
</tr>
<tr>
<td>IO(_{14})</td>
<td>0.0800</td>
</tr>
<tr>
<td>IO(_{15})</td>
<td>0.1060</td>
</tr>
<tr>
<td>IO(_{23})</td>
<td>0.0209</td>
</tr>
<tr>
<td>IO(_{24})</td>
<td>0.0028</td>
</tr>
<tr>
<td>IO(_{25})</td>
<td>0.1426</td>
</tr>
<tr>
<td>IO(_{34})</td>
<td>0.0954</td>
</tr>
<tr>
<td>IO(_{35})</td>
<td>0.1698</td>
</tr>
<tr>
<td>IO(_{45})</td>
<td>0.0111</td>
</tr>
</tbody>
</table>
attributed to processes in which magma intruded up into the Cambrian–Ordovician carbonates to form the Tongshi complex. Various types of gold deposits are distributed within the contact metasomatic zone. In the early Cretaceous, volcanic-sedimentary rocks were deposited on the Jurassic volcanic-sedimentary rocks, so part of the Tongshi intrusive complex and the contact metasomatic zone are covered by volcanic-sedimentary rocks. The gold mineralization displays spatial zonation. Porphyry gold occurrences are located within the Tongshi intrusive complex and Skarn iron–copper–gold occurrences are located in the inner contact metasomatic zone between the intrusive complex and its host rocks. Crypto-breccia and Carlin-type gold deposits are

Unit I: concealed basement; Unit II: Mesozoic volcanic sedimentary basin

Fig. 7. BIMF5 image decomposed from original gravity data for Tongshi gold field.

Unit I: concealed basement; Unit II: Mesozoic volcanic sedimentary basin; Unit IIIa: Tongshi intrusive complex; Unit IIIb: contact metasomatic zone between the Tongshi intrusive complex and its host rocks

Fig. 8. BIMF3 image decomposed from original gravity data for Tongshi gold field.
located at the outer contact metasomatic zone between the intrusive complex and its host rocks (Fig. 9). Thus, concealed gold deposits might be discovered on the northeastern side of the Tongshi intrusive complex and the contact metasomatic zone covered by early Cretaceous volcanic-sedimentary rocks.

In conclusion, BEMD was successfully applied to the decomposition of gravity data for the Tongshi gold field. The IMF3 image illustrates the spatial distribution of gold deposits more clearly than the image obtained by Fourier transform (Chen et al., 2000). The IMF3 image provides reliable evidence in the search for new gold deposits in this area. The BEMD method is not only suitable for analysis of gravity data, but also can be extended for analysis of the scattered data like magnetic data and exploration geochemical data.

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